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Comparative Evaluation of Exact Extreme Value Stochastic Flood Model for Mixed Populations.

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COMPARATIVE EVALUATION OF EXACT EXTREME VALUE STOCHASTIC FLOOD MODEL FOR MIXED POPULATIONS

A Dissertation

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by
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a = shape parameter of the Weibull distribution

a_k = shape parameter of the Weibull dist. for the k^{th} sub-population

$\hat{a}$ = sample estimate of the Weibull shape parameter a

b = scale parameter of the Weibull distribution

b_k = scale parameter of the Weibull dist. for the k^{th} sub-population

$\hat{b}$ = sample estimate of the Weibull scale parameter b

$BIAS$ = relative bias

CV = coefficient of variation

D = Kolmogorov EDF test statistic

$E(.)$ = expected value of random variable (.)

E_ν^t = $\{ \eta(t) = \nu \}$ = occurrence of ν exceedances in time interval $(0, t]$

$f(x)$ = probability distribution function (pdf) of exceedances

$F(x)$ = $P(\xi_\nu \leq x)$ = cumulative distribution function (cdf) of exceedances

$F_k(x)$ = $P(\xi_{\nu k} \leq x)$ = cdf of exceedances of the k^{th} sub-population

$F_\nu(t)$ = $P(\tau_\nu \leq t)$ = cdf of the time of occurrence of the ν^{th} exceedance

m_1 = original sample size of first flood group x

m_2 = original sample size of second flood group y

m_i = rank of i^{th} observed flood event x_i in descending order

$M_{1,0,t}$ = $M_{(t)}$ = t^{th} probability weighted moment (t = 1, 2, ...)

n = number of flood exceedances in the sample or sample size

$n_1 =$ uncensored sample size of first flood group x, truncated at Q_0

$n_2 =$ uncensored sample size of second flood group y, truncated at Q_0

$N =$ number of random samples generated for each sample of size n

$N_s =$ number of distinct sub-populations in the flood sequence

$N_y =$ Number of years of records of the observed flood sequences

$p(x_i) =$ Weibull plotting position of i^{th} observed flood event x_i

$Q_0 =$ threshold flood level of partial duration series (PDS)

$Q_\nu =$ magnitude of ν^{th} flood event that exceed Q_0

$Q_T =$ magnitude of flood event for return period of T years

$\hat{Q}_T =$ estimate of magnitude of flood event for return period of T years

$R =$ ratio of the mean and the variance of the Poisson random variable

$R_c =$ critical value of Poisson ratio R

$SE =$ relative standard error

$RMSE =$ relative root mean square error

$t =$ time period (season or year)

$U(n_1, n_2) =$ usual Mann-Whitney U statistic for flood group x and y

$U_c =$ modified Mann-Whitney U statistic

$Var(.) =$ variance of random variable (.)

$w^2 =$ Cramer-Von Mises EDF test statistic

$\bar{x} =$ mean of the observed flood events x_i

$x_i =$ observed value of i^{th} flood event in the sequence

$\hat{x}_i =$ model predicted value of i^{th} flood event with same probability as x_i

$Z_{U_c} =$ standard normal variate of modified Mann-Whitney U statistic

AFS = annual flood series

cdf = cumulative distribution function

EDF = empirical distribution function

EV1 = extreme value type 1 distribution

EV3 = extreme value type 3 distribution

FGM = flood generating mechanism

GEV = generalized extreme value distribution

i.i.d = independently and identically distributed

LN2 = two parameter log normal distribution

LP3 = log Pearson type 3 distribution

MIX = method of mix moment

MLE = maximum likelihood estimator

MMD = direct method of moment

MMI = indirect method of moment

MOM = method of moment

NERC = Natural Environmental Research Council

pdf = probability distribution function

PDS = partial duration series

POME = principle of maximum entropy

PWM = probability weighted moment

TCEV = two component extreme value distribution

USGS = United State Geological Survey

USWRC = United State Water Resources Council

$\alpha =$ statistical significance level

$\beta =$ exponential distribution parameter

$\beta_k =$ exponential parameter for the k^{th} sub-population

$\hat{\beta} =$ sample estimate of the exponential parameter β

$\theta =$ model parameter or quantile of interest

$\hat{\theta} =$ estimate of model parameter or quantile

$\hat{\theta}_i =$ estimate of θ for the i^{th} simulated sample $(i = 1, 2, \ldots, N)$

$\gamma =$ skewness coefficient

$\eta(t) =$ number flood exceedances in time interval $(0, t]$

$\eta_k(t) =$ number flood exceedances in $(0, t]$ for the k^{th} sub-population

$\lambda =$ Poisson distribution parameter (L)

$\lambda_1 =$ Poisson distribution parameter for the 1^{st} sub-population (L1)

$\lambda_2 =$ Poisson distribution parameter for the 2^{nd} sub-population (L2)

$\lambda_k =$ Poisson distribution parameter for the k^{th} sub-population

$\bar{\lambda}_j =$ j^{th} Poisson random deviate $(j = 1, 2, \ldots, n)$

$\hat{\lambda} =$ sample estimate of the Poisson parameter

$\lambda_0 =$ zeroth Lagrange multiplier

$\lambda_f =$ first Lagrange multiplier

$\lambda_s =$ second Lagrange multiplier

$\lambda(t) =$ mean rate of occurrence of exceedances in time interval $(0, t]$

$\Lambda(t) =$ $\lambda(t).t =$ mean number of exceedances in time interval $(0, t]$

$\mu =$ mean of the exceedance population (M)

$\mu_1 =$ mean of the exceedance of the 1^{st} sub-population (M1)

$\mu_2 =$ mean of the exceedance of the 2^{nd} sub-population (M2)

$\sigma =$ standard deviation of the exceedance population

$\xi_\nu =$ magnitude of the ν^{th} flood exceedance

$\xi_{\nu k} =$ magnitude of the ν^{th} flood exceedance for the k^{th} sub-population

$\tau =$ time variable for t

$\nu =$ ν^{th} flood exceedance ($\nu = 0, 1, 2, \ldots, n$)

$\phi_t(x) = P\{\chi(t) \leq x\} =$ dist. function of the largest exceedance in $(0, t]$

$\phi_k(x) = P\{\chi_k(t) \leq x\} = \phi_t(x)$ for the k^{th} sub-population

$\phi(x) =$ distribution function of the largest annual flood exceedance

$\chi(t) = sup(\xi_\nu) =$ largest flood exceedance in time interval $(0, t]$

$\psi(.) =$ psi function of (.)

$\Gamma(.) =$ gamma function of (.)

Abstract

Two alternative formulations of the exact extreme value stochastic flood model are presented in explaining the behaviour of observed flood series resulting from mixed climatological processes. As a marginal distribution for flood exceedances, a more flexible Weibull distribution is introduced in the place of the traditional exponential distribution. The asymptotic predictive performances of both the exponential and Weibull model formulations are evaluated in terms of relative bias ($BIAS$) and relative root mean square error ($RMSE$) of quantile estimation. The model with Weibull marginal performs well over the exponential for population conditions shown to exist in most of the observed flood series of Louisiana.

In identifying a robust estimator for the Weibull distribution, the predictive performances of the principle of maximum entropy (POME) and probability weighted moments (PWM) are evaluated. On the basis of $BIAS$ and $RMSE$ of quantile estimation, the POME emerges as the most robust Weibull estimator for a wide range of population conditions.

The compound formulations for both exponential and Weibull models are discussed as against the respective simple formulations in analyzing mixed flood populations. For a wide variation of the means of mixed populations, the predictive performances of both the simple and compound formulations of the exponential model are evaluated in terms of $BIAS$ and $RMSE$ of quantile estimation. Similarly, for a wide variation of the coefficient of variance of mixed populations, the performances of both formulations of the Weibull model are evaluated. The com-

pound formulations of both the exponential and Weibull models demonstrate superior performances over the respective simple formulations if statistically distinct sub-populations are present in the mixed populations.

The descriptive properties of those selected formulations of the exact extreme value stochastic flood model are evaluated on the observed flood series in Louisiana. The flood series are hydroclimatically separated and tested for the assumption of identical distribution. The flood series are also tested for the Poisson assumption in ensuring the mutual independency. The validity of the exponential and Weibull distributions as marginals are examined.

Chapter 1

Introduction

The determination of flood probability distribution functions has received considerable attention in the effort to gain a better understanding of the stochastic nature of the flood phenomenon. The stochastic flood models, based on annual flood series (AFS), tend to emphasize statistical analysis of flood data than to understand the physical mechanisms controlling the flood events. Therefore, in explaining the nature of the flood phenomenon many investigators have attempted to develop stochastic flood models based on partial duration series (PDS). Following the theory of exact extreme values (Todorovic, 1970), Todorovic and Zelenhasic (1970) derived a more general form of a stochastic flood model based on PDS in which they considered not only the occurrence of flood events but also the magnitude of those events simultaneously to describe the complex nature of the flood phenomenon. Although the general form of this model has a solid theoretical basis, the absence of statistical tools has limited its application to real data.

Most recent studies have discussed various formulations of this model with specific assumptions regarding the stochastic behaviour of the observed flood sequences in order to use the existing statistical tools which continue to appear as the 'best' approach in describing the complex flood phenomenon. The purpose of this study is to develop other alternative formulations of the exact extreme value stochastic flood model which would describe the behaviour of observed flood series resulting from mixed climatological processes. In view of this, the following model formulations are introduced and then compared in terms of both their predictive and descriptive properties to the best existing formulations:

- A more flexible Weibull marginal distribution as against the traditional exponential marginal distribution, and
- The compound formulation for hydroclimatically separated mixed flood populations as against the simple formulation for mixed flood populations.

In previous studies to describe the distribution of flood exceedances, the exponential distribution has often been used as the marginal in the exact extreme value model (exponential model) due to its mathematical simplicity. As an alternative to the traditional exponential marginal, the Weibull distribution can be used to describe flood samples demonstrating much wider variability (Hirschboeck and Cruise, 1990). Therefore, in this study the feasibility of the Weibull distribution which contains the exponential as a special case is examined as the marginal in the exact extreme value stochastic flood model (Weibull model). Singh et al. (1990) discussed the comparative performance of various estimators in estimating the Weibull parameters and quantiles. Following the results of their study, the asymptotic properties of the POME and PWM estimators for Weibull quantile estimation are analyzed for population conditions believed to exist in the observed flood series of Louisiana.

In recognizing the fact that many observed flood sequences are drawn from mixed populations, various studies have taken different approaches to identify and separate those flood events into homogeneous sub-populations. Some studies have developed mathematical methodologies, while others have made a seasonal approach to identify and separate flood events into homogeneous sub-populations. Following those studies, Hirschboeck (1985, 1987) explained a more detailed hydroclimatic separation approach to separate mixed populations by identifying the physical mechanisms that generate each flood event in the flood sequence. In considering the behaviour of the observed flood sequences in Louisiana it is assumed that two distinct flood generating mechanisms produced all the important flooding. These are frontal activities occurring during winter/spring and events related to convectional thunderstorms and tropical disturbed weather. occurring during summer/fall (USGS, 1988). In this respect a somewhat simplified hydroclimatic separation approach is used in this study.

In selecting a flood-like probability distribution, the asymptotic predictive properties of both the simple and compound formulations of the exponential and Weibull models are analyzed in terms of statistical characteristics measured by performance indices such as $BIAS$ and $RMSE$ of quantile estimation. Following the comparative analysis of predictive properties of those stochastic flood models, the descriptive properties of the same are examined for the observed data as realistic population conditions. Unlike simulated data, which are error-free as they can be generated according to any given probability distribution, the observed data are tested for underlying model assumptions before examining the descriptive properties of those stochastic flood models.

The Poisson statistical test, described by Cunnane (1979) is used to ensure the mutual independence assumption for the observed flood series. In examining the assumption of identical distributions of observed flood series, hydroclimatically separated flood groups are tested for statistical similarities using the modified Mann-Whitney U statistical test. When using the relatively inflexible one parameter exponential marginal, the Kolmogorov(D) and the Cramer-Von Mises (w^2) tests, introduced by Stephens (1974) are used to test the validity of the exponential assumption for the marginal distribution of flood exceedances.

Chapter 2

Literature Review

2.1 Introduction

A brief survey of the historical development of flood frequency studies during the period late 1890 and early 1960 was reported by Chow (1965). Greis (1983) reviewed the historical development of flood frequency studies following the United States Water Resources Council (USWRC) uniform approach to flood frequency analysis (Bulletin 15, 1967). Kirby and Moss (1985) have investigated the principal aspects of the American practice of flood frequency studies since 1860's. In 1985, Lettenmair and Potter reviewed the recent developments in regional flood frequency analysis. Cunnane (1986) has discussed the historical development of selected topics in flood frequency analysis including at-site and regional flood estimates, predictive and descriptive performances of parameter estimators and regional homogeneity and other statistical behaviour of flood magnitudes.

In the following literature survey, an attempt has been made to review the historical development of partial duration series flood models and some related topics which are directly relevant to the proposed research. In addition, a brief review of the historical development of hydrologic flood frequency analysis and annual duration series flood models has been made as those are indirectly relevant to the present study.

2.2 Hydrologic Flood Frequency Analysis

The probability approach to flood frequency analysis started in early 1900's when the Gaussian normal probability law was introduced as the basic tool to describe the observed flood series (Horton, 1913). Later, Hazen (1914) showed that the usually skewed flood probability distributions can be better represented by the Galton log-probability law, instead of the Gaussian law. Foster (1924) who preferred to work with untransformed flood data introduced the Pearson type 3 probability distribution function and expected it to fit any skewed observed flood series. Tippett (1925) was one of the first to determine the probabilities associated with extreme values taken from a normal initial distribution. Immediately following this, Frechet (1927) and Fisher and Tippett (1928) derived the 'asymptotic extreme value theory' for extreme values taken from a given initial distribution. Gumbel (1941) applied this theory to flood frequency analysis by defining the initial distribution as exponential type and by having no upper bound on extreme flood events. This is the well known Gumbel's extreme value type 1 (EV1) distribution.

Following these numerous flood probability distribution models, the USWRC recommended the use of the log Pearson type 3 (LP3) distribution as a uniform approach to flood frequency analysis for U.S. flood series (Bulletin 15, 1967). However, even with a follow-up series of analytical and statistical refinements (Bulletin 17, 1976; Bulletin 17a, 1977 and Bulletin 17b, 1981) the USWRC uniform approach remains controversial from a statistical standpoint. As a result, research in the field of flood frequency analysis has rapidly developed into many other related areas.

2.3 Hydrologic Flood Models

In general, many of the original flood data have little significance in the flood frequency analysis because the maximum design flood is usually determined by a few critical observations, i.e. large floods. Therefore, the hydrologic flood data are generally extracted from their original complete duration series and presented either in the form of annual flood series (AFS) or partial duration series (PDS). However, from the standpoint of the theory of statistical modeling, any array of flood events based on either the AFS or PDS should be required to form a pure-random sample in order to develop a statistically efficient stochastic flood model.

Cunnane (1973) discussed the statistical efficiency in terms of variance of quantiles estimates of AFS and PDS flood models based on observed flood data. In that study, he concluded that the flood frequency models based on PDS are statistically efficient in estimating flood quantiles when the flood series contains more than 1.65 flood exceedances per year. Tavares and De Silva (1983), using Monte-Carlo simulation, compared the statistical efficiencies between the AFS and PDS models in terms of variance of flood quantile estimates and found that the PDS model has significantly lower estimation variance than the AFS model when it contains more than twice as many as flood exceedances per year. This deviation in observations among the above studies, in which one is based on a theoretical variance formula (Cunnane, 1973) and the other is based on Monte-Carlo simulation (Tavares and De Silva, 1983) has been explained by Rosbjerg (1985). When explaining this deviation, Rosbjerg (1985) considered both statistically independent and dependent cases in developing a formula for the variance of the flood quantile estimates. Accordingly, he concluded that the advantage of the PDS over the AFS flood models is strongly dependent upon the statistical independence assumption of flood exceedances. Ashkar et al. (1987) have analyzed the statistical efficiencies of PDS models in terms of both the number of exceedances per year and the sample sizes. In this study, they found that the PDS models can be more efficient than the AFS models, particularly for small samples (< 30) even when the number of exceedances per year in PDS are lower than the unity.

2.4 Annual Flood Series Models

An annual flood series (AFS) is a sequence of floods with the annual flood defined as the maximum instantaneous peak discharge of each water year. Sometimes the maximum mean daily discharge of each year is used as the annual flood. There are many problems encountered with respect to the use of annual floods in stochastic flood models. First, the AFS uses only a limited number of events to describe the flood phenomenon as it considers only the largest flood and ignores the other largest floods in a given year. Second, the annual floods in some dry years are so small yet they can even qualify as floods. Finally, the stochastic flood models based on AFS tend to stress more the statistical analysis of flood data than the understanding of the physical mechanisms controlling the flood events. Despite these problems, a large number of AFS based stochastic flood models have been developed.

Following the asymptotic extreme value theory, Gumbel (1941) hypothesized that the flood events of an AFS followed the EV1 distribution. In applying the asymptotic extreme value theory, Gumbel assumed that the number of floods in a year is indefinitely large (asymptotic condition) and those floods are statistically independent, identical and exponentially distributed. Unfortunately, these critical assumptions are not always supported by the properties of observed flood series. Hence, the theoretical arguments that annual floods follow an extreme value type 1 (EV1) distribution are not very convincing. However, from a statistical standpoint, both parameters of EV1 can be easily estimated and the explicit inverse closed form of the cumulative distributed function (cdf) of EV1 provides a mathematically efficient estimation of flood quantiles. Chow (1954) discussed the limitations of the EV1 distribution with respect to its constant skewness (1.14) and introduced the two parameter log normal (LN2) distribution into flood frequency analysis. The proposed LN2 distribution was made to offer a theoretical interpretation towards the reduction of skewness and the treatment of random effects in annual data.

Many years after the introduction of the Pearson type 3 distribution (Foster, 1924), the USWRC recommended the use of the LP3 distribution (Bulletin 15,

1967) as a uniform approach to flood frequency analysis of AFS in the U.S. Since then, this approach has been subjected to a series of analytical and statistical refinements (Bulletin 17, 1976; Bulletin 17a, 1977 and Bulletin 17b, 1981) in improving the accuracy of flood quantile estimates. Such refinements are designed to minimize the bias of expected mean, to provide methodologies in adjusting for zero flows, incomplete data and outliers and most importantly to compute reliable estimate of skewness.

The general disagreement with the uniform approach, mainly due to its instability in skew estimates, initiated considerable research into other related areas of flood frequency analysis. The results of simulation studies made by Matalas et al. (1975) have suggested that the concept of regionalizing the skewness of AFS should be used with caution. Klemes (1976) made further cautions in this regard based on hydrological grounds such as basin physiographic features. Bobee (1975) and Bobee and Robitaillie (1977) made comparative evaluations of different estimators of LP3 distribution based on AFS. They concluded that the direct method of moment (MMD) provided a better fit to the data than the indirect method of moment (MMI) recommended by USWRC. Rao (1980) suggested a new estimator, the method of mix moments (MIX) and found it superior to the other methods (MMD, MMI) based on simulation studies.

In addition to different estimators of LP3 distribution, many other probability distributions have been introduced to the flood frequency analysis of AFS. The Natural Environment Research Council (NERC, 1975) adopted the generalized extreme value (GEV) distribution as the standard approach to follow in the United Kingdom flood series. The Wakeby distribution (Houghton, 1978) and the Two Component Extreme Value (TCEV) distribution (Rossi et al., 1984) are some of the recently developed AFS flood models which are capable of modelling the positively skewed or heavy tailed distributions.

2.5 Partial Duration Series Flood Models

A partial duration series (PDS) is a sequence of floods which are selected above a certain base or threshold level of discharge. In the early stage of hydrologic practice, an arbitrary criterion was used in choosing the threshold level, such that on the average two to three flood exceedance events were included for each year (Langbein, 1949). This additional flood information, provided by the PDS is considered to be important in design of many hydraulic structures. For instance, the design of bridge piers, culverts and storm sewers are governed by the frequency and magnitude of flood events exceeding a certain threshold level. On the other hand, the AFS which contains only the extreme flood events can be used in some hydrologic designs such as spillway structures and flood plain analysis.

The probabilistic relationship between AFS and PDS was first investigated by Langbein (1949). Later, Chow (1950) derived a theoretical relationship between the probabilities of AFS and PDS, in which he concluded that the probability estimates of magnitudes of both series do not differ much except for low magnitudes. However, construction of a feasible stochastic flood model for the AFS is restricted by lack of solid theoretical and physical grounds upon which such a model could be based. Therefore, the majority of recent flood models developed for PDS have been designed to follow theoretical and physical bases in explaining the nature of the flood phenomenon. Holl and Howell (1963) discussed the probability that a flood event of a certain magnitude will be equaled or exceeded at least once in a specific time period, assuming the occurrence of flood events as a time independent Poisson process. Borgman (1963) and Shane and Lynn (1964) developed several measures of risk criteria for design discharge assuming a time independent Poisson process for the occurrence of flood events in a PDS. Some of the measures of risk criteria derived were the encounter probability, the distribution of waiting times and the expected recurrence interval. Kirby (1969) discussed the distribution process of the occurrence of major flood events in the PDS. He considered flood events as exceedances in a sequence of randomly spaced Bernoulli trials, representing the occurrence of flood hydrograph peaks which are arbitrarily classified as floods. Further, he showed that the probability distribution of occurrence of flood events approaches a time independent Poissonian process for sufficiently small exceedance probabilities.

Todorovic (1970) presented a new approach to analyze extreme values of random numbers of random or naturally occurring phenomena. Although the basic concept behind this approach is similar to that of the 'asymptotic extreme value theory', important fundamental differences exist. In his study, Todorovic used the 'exact extreme value theory' for extreme events taken from a variable random sample of which the magnitudes of events are not necessarily identically distributed. Following this theory, Todorovic and Zelenhasic (1970) have developed a more 'general' form of 'exact extreme value stochastic flood model' based on PDS in which they consider not only the occurrence of flood events but also the magnitude of those events, simultaneously. However, this general form of a distribution function ($\phi_t(x)$) of the largest exceedances ($\chi(t)$) in a given interval of time $(0, t]$ was simplified to a 'particular' form to solve a specific problem. This particular form considered the following assumptions: *a*) the occurrence of exceedances ($\eta(t)$) is a time dependent Poisson process, *b*) the exceedances (ξ_ν) are independent and identically distributed (i.i.d.), *c*) the two sequences, the exceedances (ξ_ν) and the time of occurrence of exceedances (τ_ν) are stochastically independent, and *d*) there exists a common distribution function ($F(x)$) of all exceedances in exponential form within a specific time interval $(0, t]$.

Todorovic and Rousselle (1971) have extended the particular form of exact extreme value flood model to non-identically distributed exceedances on the assumption that events are identically distributed during a particular time period, (e.g. a season or a month). Todorovic and Woolhiser (1972), using the same assumptions as in the particular form of exact extreme value flood model, derived an expression for the joint-distribution function of the largest exceedances ($\chi(t)$) and its time of occurrence ($T(t)$). Gupta et al. (1976) obtained an expression for the joint-distribution function under the same assumptions as Todorovic and Rousselle (1971). Todorovic (1978) derived somewhat generalized forms of expressions for

$\phi_t(x)$, under the assumption of mutual independency between the sequences of random variables ξ_ν and τ_ν. Further, he determined the distribution functions for the volume of the ν^{th} exceedance and the largest volume within a given time interval $(0, t]$.

Ashkar and Rousselle (1982), using the exact extreme value flood model, derived a multivariate distribution function for flood exceedances, volume and duration. Rosbjerg (1987a) derived a bivariate exponential distribution in the Marshall-Olkin form and discussed the stochastic dependency of ξ_ν and τ_ν. In recent years, Singh and Rajagopal (1987) presented a methodology to determine the bivariate distribution function for ξ_ν and τ_ν, using the Finch-Groblicki's 0-marginal concept. They used the Kulback-Leibler information criteria to specify a proper 0-marginal function and thereby, unique bivariate distribution function. Singh and Singh (1991) used the same concept to determine bivariate distribution functions for various forms of 0-marginal functions and selected the one which fits the best to observed flood series.

2.5.1 Selection of Threshold Level

When using the stochastic flood models based on PDS, the selection of a threshold level above which streamflows are regarded as floods plays a major role in terms of assumptions made on ξ_ν as independent events and $\eta(t)$ distributed as a Poisson process (Todorovic and Zelenhasic, 1970). Cunnane (1979) derived a statistical criteria to test the distribution of $\eta(t)$ for the Poisson assumption in excess of a threshold level. This statistical criteria is derived on the basis of the equality of the mean and variance of the Poisson distribution. He further stated that in obtaining a point estimation of flood magnitude, the distribution of $\eta(t)$ does not play a role, but in estimating confidence limits of flood magnitude the distribution of $\eta(t)$ is important. Miquel and Bernier (1981) suggested the negative binomial distribution as an alternative to the Poisson distribution for $\eta(t)$, when the variance is greater than the mean.

Kavvas (1982) and Cervantes et al. (1983) studied the clustering phenomenon in flood series in order to avoid the assumptions that the ξ_ν are i.i.d. and $\eta(t)$ follows a Poisson process. In doing this, they treated the threshold level in the PDS as a random variable, which allows the threshold level to be considered as a function of hydraulic structure. Therefore, these cluster models may be particularly applicable to low threshold levels where the flood exceedances and the occurrence process have a stochastically dependent structure. In their review of cluster models, Rossi et al. (1984) stated that the Kavvas models still can be considered as a particular form of Todorovic's general form of exact extreme value flood models when a flood is defined as the largest value within each cluster.

An interesting discussion has been made by Ashkar and Rousselle (1983) on the choice of selection of threshold level. In this study, they showed that once the PDS has been accepted as a Poisson admissible process for a certain threshold level, then it remains so for any higher threshold level. Cruise (1986) has discussed the effects on flood quantile estimation of threshold level selection through Monte-Carlo simulation. Ashkar and Rousselle (1987) presented a technique (R-curve criteria) for threshold level selection as a continuation of Cunnane's work (1979) and showed that a strong statistical independency exists between the extreme events when the PDS is Poisson admissible.

The Poisson admissible exact extreme value stochastic flood model has been used in various hydrologic applications. Ouellette et al. (1985), El-Jabi and Rousselle (1987) and Leblanc and Ouellette (1988) have used this model in flood damage and zoning analysis. Cruise and Singh (1987, 1988) have applied this model for streamflow waste release analysis. In particular to this study, a strategy for formulating and testing the Poisson exact extreme value flood model presented by Cruise and Arora (1990) using the observed flood series in Louisiana, is closely followed.

2.5.2 Heterogeneity of Flood Exceedances

When developing a valid stochastic flood model, the assumption that flood exceedances belong to an identically distributed (homogeneous) population has played a major role. Therefore, it is important to examine the presence of mixed populations in streamflow data. Potter (1958) was one of the first to discuss the evidence of mixed (nonhomogeneous) populations in annual flood series. Further, he outlined some possible climatic causes for mixed populations without exploring them in detail. In recognizing the fact that many observed flood sequences are drawn from a mixture of populations, a number of investigators have developed various forms of mixed distribution stochastic flood models. When developing mixed distribution models, different approaches were used to identify and separate flood exceedances into homogeneous sub-populations.

Hasselblad (1969), Singh (1974) and Varsace et al. (1982) discussed mathematical methodologies for separating two exponentially distributed flood exceedances of sub-populations of a given flood sequence. Although they recognized the role of climate in producing mixed populations in streamflow data, the principal approach behind those methodologies was to objectively identify flood exceedances from a statistical perspective and hence to define mixed distributions. The studies made by Todorovic and Rousselle (1971), Guillot (1973), Nachtnebel and Konecny (1987) and Diehl and Potter (1987) identified and separated seasonally homogeneous sub-populations of a given flood sequence. This seasonal separation identifies the seasonal climatic variability and its influence on flood sequence homogeneity. To this effect, Cruise and Singh (1987), USGS (1988) and Cruise and Arora (1990) have discussed that two distinct flood generating climatic mechanisms produced all of the important flooding in winter and summer seasons in Louisiana.

Jarrett and Costa (1982) and Waylen and Woo (1982) used physical evidence to identify flood generating processes of rainfall and snowmelt and thereby to separate the flood sequence into rainfall and snowmelt sub-populations. Hirschboeck (1985, 1987) explained a more detailed hydroclimatic approach to separate sub-populations of a flood sequence by identifying various synoptic atmospheric circulation mechanisms and patterns that generated each flood event in the flood sequence.

2.5.3 Marginal Distribution of Exceedances

The other important assumption that requires investigation is the marginal distribution of the magnitude of all flood exceedances for a given time interval $(0, t]$. Zelenhasic (1970) discussed the use of a family of two-parameter gamma distributions (in exact extreme value flood models) as the marginal distributions for exceedances in deriving theoretical distributions of the largest exceedances ($\phi_t(x)$). However, when using the observed flood data to estimate the distribution function of $\phi_t(x)$, only the exponential distribution which is a particular case of gamma distribution was considered as the marginal distribution due to its closed form nature of its inverse function of cdf and presence of one parameter.

Rosbjerg (1987b) and Cruise and Arora (1990) noted that in some observed flood sequences, it is necessary to establish a high threshold level to meet the exponential hypothesis for the marginal distribution. This high threshold level may censor many of the events in the sequence, resulting in unreliable estimates of extreme events. Therefore, Rosbjerg (1987b) proposed the LN2 distribution as an alternative to the exponential marginal. However, the application of the LN2 distribution as the marginal has the following disadvantages: *a*) it cannot be used satisfactorily to model many observed flood series with relatively large skewnesses produced by mixed climatic processes, *b*) it preserves the statistical properties of log space values of observed data instead of real space values, and *c*) nonexistence of closed form inverse function for the cdf. Cruise and Arora (1990) have discussed the sensitivity of quantile estimates with the exponential as the marginal to lower threshold level selection. They suggested that it may be advantageous to maintain a relatively low threshold level (as low as Poisson admissible) when it is difficult to meet the exponential hypothesis or when mixed populations are present in the observed flood sequence.

Therefore, in the proposed research the Weibull distribution, which contains the the exponential as a special case, was examined as the marginal distribution. The Weibull distribution is an extreme value type 3 (EV3) distribution, in which the left tail is bounded by zero. Therefore, this distribution is often used to analyze minimum type extreme events in hydrology as well as flood frequency analysis. Gumbel (1954) first applied the Weibull distribution for hydrologic drought frequency analysis. Grace and Eagleson (1970) and Rao and Chenchayya (1974) used the Weibull distribution to analyze the durations of wet and dry sequences. Singh (1987) derived the Weibull distribution, using the principle of maximum of entropy (POME) and compared its estimates of rainfall depths and durations against the estimates of the same by MOM and MLE.

2.6 Parameter Estimation

Several methods have been proposed for estimating parameters of various probability distributions used in hydrologic practice. Traditionally, the most popular methods such as method of moments (MOM), maximum likelihood estimation (MLE) and methods of least squares (MLS) have been widely used in estimating parameters of a given probability distribution. Some of the other recently derived estimation methods that have been widely used are probability weighted moments (PWM) introduced by Greenwood et al. (1979) and principle of maximum entropy (POME) derived by Jaynes (1961, 1982) based upon the concept of entropy (Shannon, 1948a and 1948b). However, interaction between a particular probability distribution and the selected method of parameter estimation led to an evaluation of under what conditions one parameter estimation method is superior to another. The certain statistical performance indices such as relative bias ($BIAS$), relative standard error(SE) and relative root mean square error($RMSE$) of estimated parameters and quantiles are often used for comparing different parameter estimation methods for a given probability distribution.

In this study, the results of comparative evaluations of different methods in estimating parameters of the Weibull distribution reported by Singh et al. (1990) are closely followed. They investigated the statistical performances of various estimators in estimating parameters and quantiles of the Weibull distribution by Monte-Carlo simulation and concluded that MLE and POME estimators perform most consistently in terms of $BIAS$ and $RMSE$ for a wide range of population parameters. Further, they stated that PWM estimator performs well where large variance is expected in the population.

2.7 Monte-Carlo Sampling Experiments

A problem of continuing interest in hydrologic flood frequency analysis is that of obtaining a reliable estimate of flood quantiles. In general, such estimates are subject to both 'model' and 'sampling' errors which thereby limit the accuracy of predictions of stochastic flood models. In addressing this problem, Monte-Carlo based sampling experiments are often used to assess the relative model performances in terms of certain statistical indices. Most of the early studies based on Monte-Carlo sampling experiments have been designed to address only the relative performances of different 'at-site specific' stochastic flood models and their estimators.

2.7.1 Performances of At-Site Flood Models

Benson (1952) and Nash and Amorocho (1966) used Monte-Carlo based simulated data distributed in accordance with an EV1 population mainly to study the standard errors of flood quantile estimates for varying sample sizes. In evaluating the performances of a number of estimators in terms of bias and efficiency, a series of sampling experiments have been carried out for the EV1 distribution by Lowery and Nash (1970) and for the Pearson type 3 distribution by Matalas and Wallis (1973) who found that next to the MLE, the MOM has performed comparatively better. Wallis et al. (1974) used sampling experiments to describe the random sampling behaviour of commonly used statistics such as mean (μ), standard deviation (σ), and coefficient of skewness (γ) for samples derived from various distributions including LP3 and EV3 distributions. They found that the estimates of γ are not only subject to large sampling errors but are biased and bounded.

In comparing the relationship between the mean and the standard deviation of regional estimates of the coefficient of skewness of historic flood sequences to that of the Monte-Carlo generated samples derived from most commonly used distributions, Matalas et al. (1975) observed the 'separation effect' of skewness of historic flood sequences. While explaining why commonly used distributions have failed to account for the separation effect of skewness, Houghton (1978) introduced the Wakeby distribution. Random samples were generated accordingly and thereby accounted for separation effect. In explaining the separation effect or the variability of skewness of observed flood series, Versace at el. (1982) and Rossi et al. (1984) introduced the two component extreme value (TCEV) distribution to describe mixed flood populations generated by different climatic mechanisms. Following the USWRC uniform approach to flood frequency analysis, Landwehr et al. (1978) extended the Matalas et al (1975) work by generating random samples in accordance with LP3 distribution and concluded that as in the uniform approach, the use of regional skew maps on log space are counter-productive.

2.7.2 Performances of Regional Flood Models

In the early 1980's, most of the research topics based on sampling experiments have concentrated on statistical properties and performances of 'regional' stochastic flood models. Greis and Wood (1981) based on Monte-Carlo generated random samples distributed in accordance with EV1 distribution, showed that the PWM regional estimates of at-site flood quantiles have the lowest mean square errors, when compared to that of the more conventional MLE and MOM. Accordingly, Stedinger (1983) suggested that if log space of flood events is used, the PWM regional estimates of at-site flood quantiles can be further improved. Lettenmaier and Potter (1985) generated random samples based on EV1, LN2 and LP3 distributions to describe a regional flood model in which the flood statistics were considered to be dependent on drainage area. Unlike the USGS index flood method, this regional model has been parameterized in terms of coefficient of variations and found to perform well only for low values of coefficient of variation.

Following the introduction of TCEV distribution (Rossi et al., 1984), Beran et al. (1986) have discussed the statistical properties of TCEV distribution based on both observed U.K. flood data and Monte-Carlo generated samples and found that the TCEV distribution is not only suitable as a regional flood model but also capable of demonstrating the separation effect described by Matalas et al. (1975). Arnell and Beran (1987) carried out a series of simulation experiments to compare the robustness of the regional TCEV estimation procedure against the other index type regional estimators, including the regional PWM estimators of GEV and Wakeby distributions. They found in terms of bias TCEV performed well, but in terms of variance the Wakeby distribution was more successful. Fiorentino et al. (1987) compared the POME estimates of regional TCEV distribution to that of the MLE estimates based on sampling experiments and found that the POME estimation procedure is relatively simple and the $RMSE$ of flood quantile estimates exhibit favourable results.

As it appears in these numerous studies, the Monte-Carlo based simulated data can be used to study the predictive abilities of stochastic flood models for a range of realistic population conditions. Thereby, the most 'robust' stochastic flood model (or models), which has the ability to predict its estimates with minimum bias and maximum efficiency can be selected (Cunnane, 1987). In evaluating the descriptive abilities of stochastic flood models, the fittings of the selected flood models can be compared on observed flood data. Cunnane (1987) discussed a criterion for selecting the 'best' stochastic flood model in terms of both predictive and descriptive properties suggesting that these properties should not be compared but should be used as complementary to each other.

Chapter 3

Stochastic Modeling of Flood Process

3.1 Introduction

The objective of this research is to develop a stochastic flood model for flood frequency analysis of PDS streamflow sequences resulting from mixed climatological mechanisms. The basic theory of the proposed exact extreme value stochastic flood model was originally developed by Todorovic (1970) and Todorovic and Zelenhasic (1970). Following them, this stochastic model has been extensively discussed and applied in its various forms in flood frequency analysis of PDS. The form of the proposed stochastic flood model is designed to use hydroclimatically defined streamflow sub-populations (Hirschboeck, 1985) and to consider a parismonious marginal distribution. This chapter will be concentrated on the description of the problem and the development of the proposed exact extreme value stochastic flood model and estimation of its parameters by the POME and PWM.

3.2 Description of Problem

Mixed Populations:

In traditional flood frequency analysis, homogeneity in the magnitudes of the flood population is considered as an important underlying assumption for the stochastic modeling of the flood process. However, evidence of mixed populations in observed flood sequences was noted by Potter (1958) as seen in 'dog leg' shape flood frequency curves. Following the discussions of many researchers (Hasselblad, 1969; Todorovic and Rousselle, 1971; Guilliot, 1973 and Singh, 1974) on physical processes of initiating floods, the USWRC Bulletin 17b (1981) discussed the presence of mixed populations in observed flood data generated by different climatic processes such as rainfall and snowmelt, etc.

Versace et al. (1982) and Rossi et al. (1984) introduced the TCEV distribution as a mixture of two exponential distributions to analyze the presence of mixed populations as basic and outlying components in AFS arising from normal and extreme flood producing storms respectively using Italian flood data. Unlike most common pdf's, the TCEV distribution was able to reproduce the statistical characteristics of flood sequences, in particular the behaviour of the right tail of the pdf and hence the variability of skewness of observed AFS within a geographically homogeneous region. They showed that the high variability of skewness of Italian flood data resulted from the presence of isolated large flood events generated by extreme climatic conditions which qualified as outliers under the EV1 distribution.

As shown in Figure 3.1, the same analysis was made by Hirschboeck and Cruise (1989) using sixteen AFS of Louisiana flood data. Their results demonstrated the usefulness of the mixed distribution approach where mixed populations of observed flood data are present in a particular hydrologic region. They further used the same regional flood data divided into non-overlaping 30 year periods and computed coefficient of skewness of each 30 year period as described by Matalas et al. (1975). Then the mean and standard deviation of these skew coefficients were compared to the same of several common single pdf's given by Matalas et al. (1975) as shown in Figure 3.2. Thus, it is clear that these single pdf's such as EV1, LN, LP3, Pareto, and Weibull failed to account adequately for the variability of skewness of Louisiana flood data. Therefore, these studies have indicated the necessity of the mixed distribution approach where flood events are generated by different climatic processes such as fronts, tropical storms and convective thunderstorms as in the case of Louisiana regions.

The typical flood hydrographs resulting from these different climatic processes shown in Figure 3.3 (Hirschboeck, 1987) demonstrate the possible presence of non-homogeneous flood populations in observed flood data and hence indicate the use of the mixed distribution approach may be justified. The hydroclimatic separation procedure described by Hirschboeck (1985, 1987) determines the climatic origins of individual flood events in a given flood sequence and then groups those events produced by the same flood generating mechanisms into hydroclimatically similar flood groups.

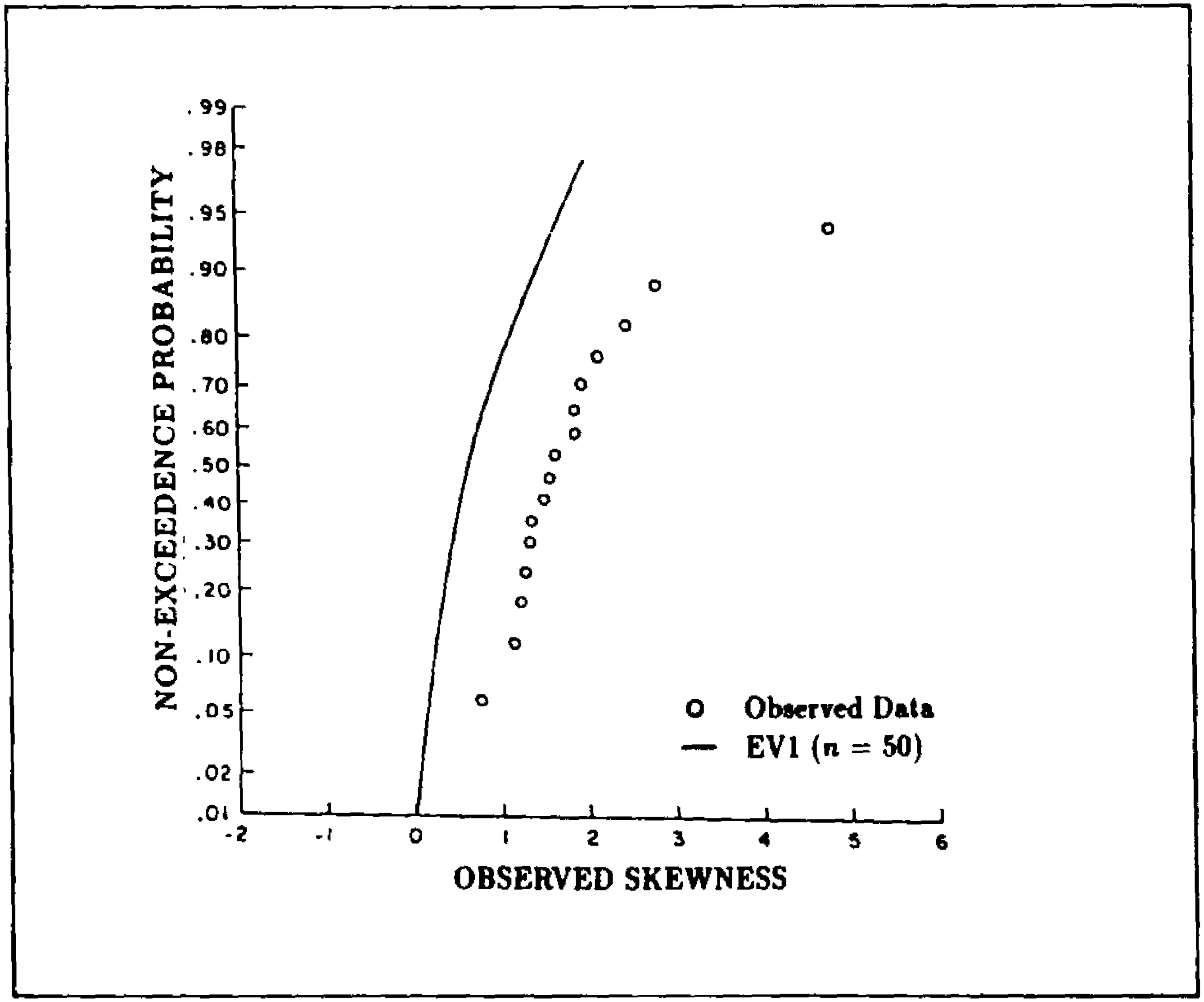

Figure 3.1 Observed cdf of the Skewness of Louisiana AFS's within a Geographically Homogeneous Region (Cruise and Hirschboeck, 1989).

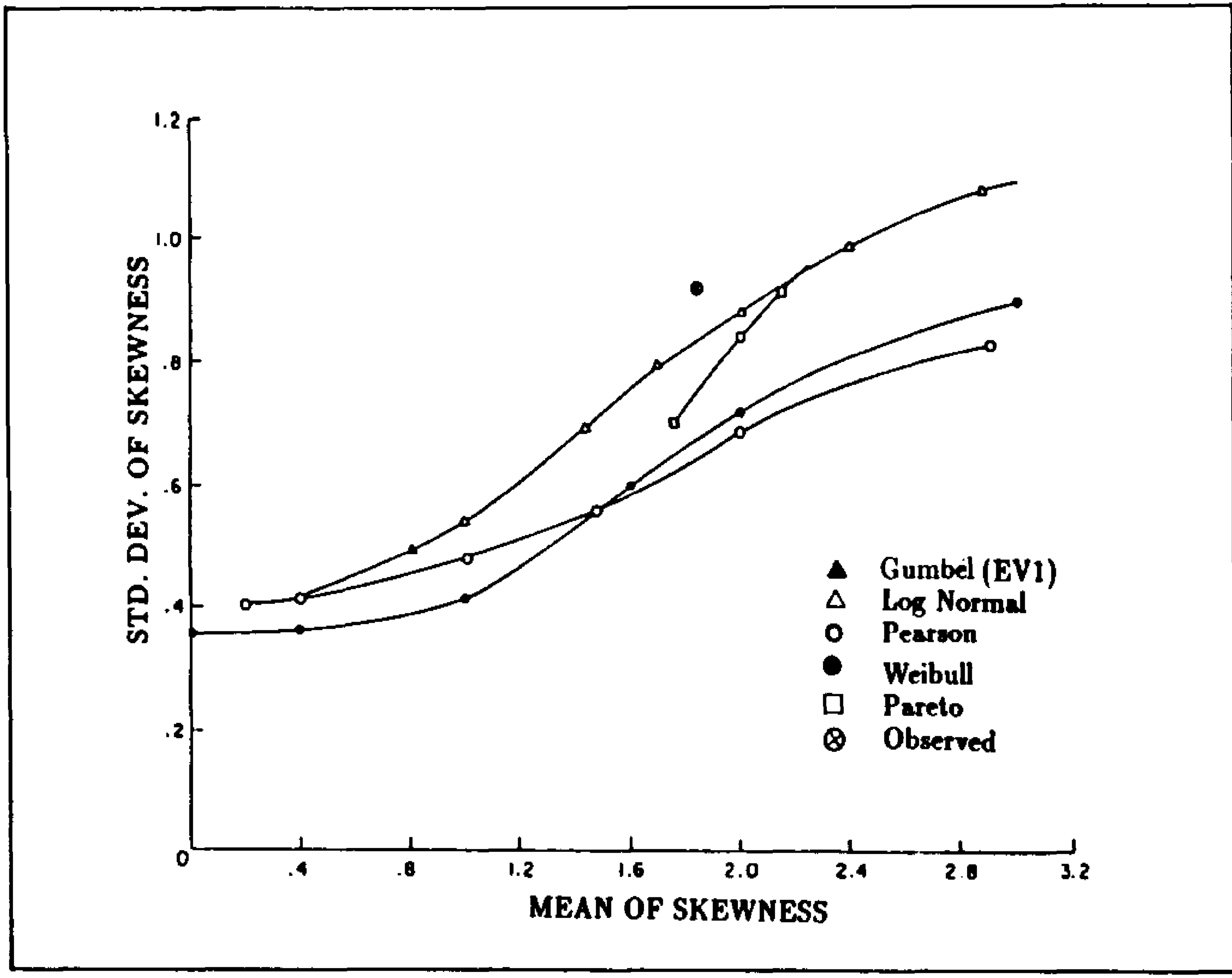

Figure 3.2 Mean Vs Standard Deviation of Skewness for $n=30$, Observed and Simulated (EV1, LN, LP3, Pareto and Weibull) Data (Cruise and Hirschboeck, 1989).

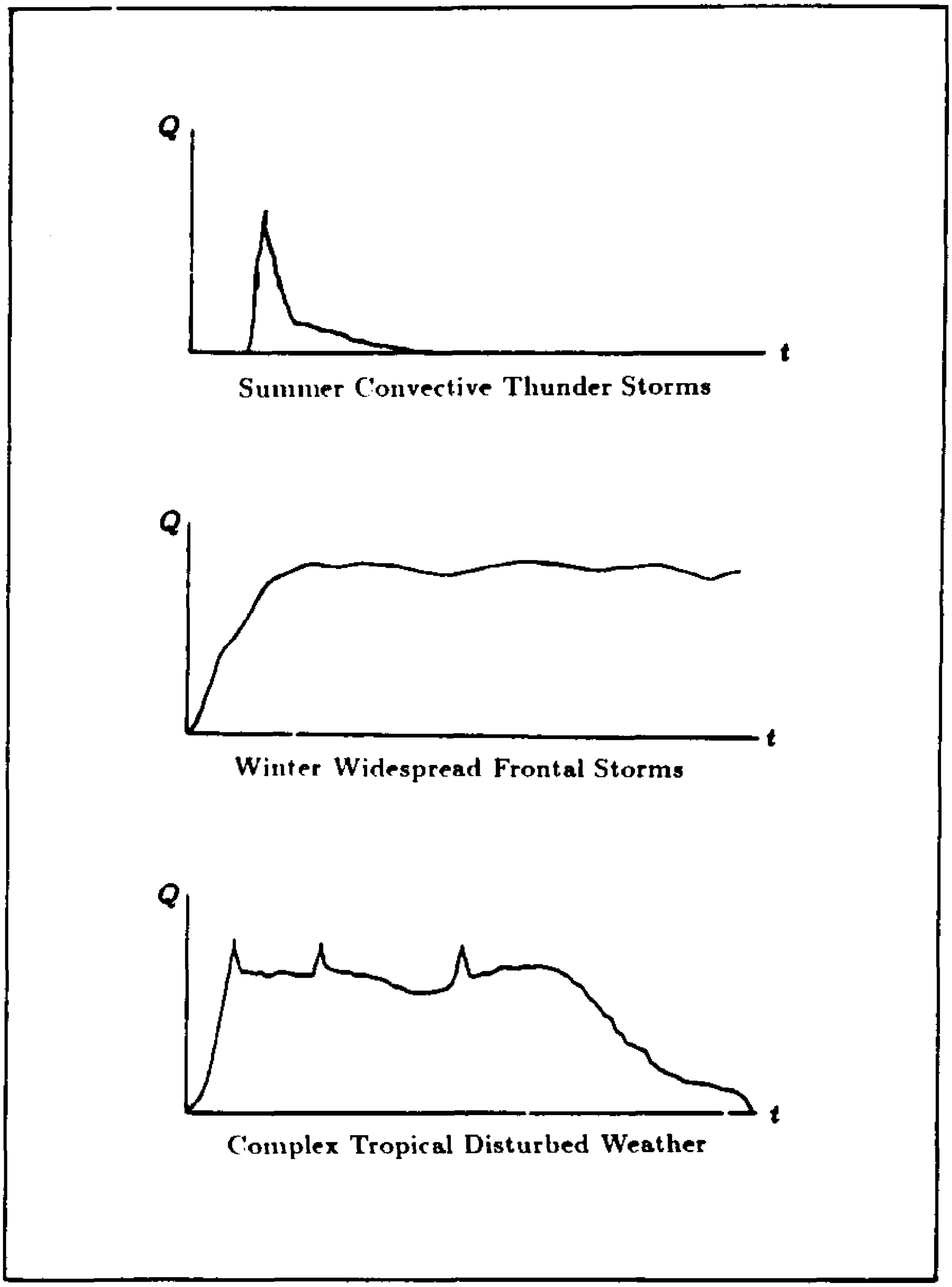

Figure 3.3 Typical Flood Hydrographs Generating from Different Atmospheric Mechanisms (Hirschboeck, 1987).

Hydrologic Flood Models:

In understanding of the theoretical and physical base of the controlling mechanisms of observed flood events, the PDS and cluster based models are often more favourable than the AFS models. However, in describing the physical processes of flooding using cluster based models, the inclusion of additional processes (ex. rainfall) as the flood generating mechanism (FGM) requires the consideration of bivariate type stochastic flood models (2 - dimensional cluster models). Therefore, cluster models are often recommended in describing natural processes such as rainfall and earthquakes which requires only univariate type stochastic models (1 - dimensional cluster models). In their review of cluster models, Rossi et al. (1984) noted that these models can still be considered as a particular class of PDS based exact extreme value stochastic flood models, if a flood is defined as the largest value within each cluster. Thus, a more generalized exact extreme value stochastic flood model is proposed to describe flood events in a PDS evolved from different climatic processes.

Poisson Assumption:

In deriving the exact extreme value stochastic flood model, the Poisson distribution has often been used to describe the process of occurrence of flood exceedances. However, Cunnane (1979) showed that the same model can still be derived without the Poisson assumption for distribution of occurrence of flood exceedances. He further stated that this Poisson assumption is only necessary when obtaining interval estimates of flood magnitudes but not in obtaining point estimates of the same.

According to Ashkar and Rousselle (1987), Poisson admissibility of the process of occurrence of flood exceedances can be used to ensure the statistical independency between extreme flood events of PDS. Therefore, in the proposed model the USGS threshold levels of the observed PDS may be used initially to obtain relatively large samples and thereby to minimize the sampling errors. In addition, at this stage of the analysis, the Poisson hypothesis could be tested to ensure the statistical independency of extreme flood events.

Marginal Distributions:

In traditional exact extreme value stochastic flood models, the exponential[1] distribution has often been considered as the marginal distribution of the magnitude of flood exceedances due to its closed form inverse function of cdf and the presence of one parameter. The exponential distribution has coefficient of variation (CV) equal to unity ($CV = \sigma/\mu = \beta^{-1}/\beta^{-1} = 1$); thus, when fitting to a given flood sample, the Poisson accepted threshold level which defines the PDS may need to be raised until the CV approaches unity in order to meet the exponential hypothesis. This raising of the threshold level may reduce both the observed variability of skewness and the sample size and thereby increase the sampling error.

Cruise and Arora (1990) have suggested that it may be advantageous to maintain a relatively low threshold level (as low as Poisson admissible) in the model when it is difficult to meet the exponential hypothesis, or in the presence of mixed populations in the PDS. Therefore, a parismonious marginal, the two parameter

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[1] β - exponential parameter

Weibull distribution is introduced into the proposed stochastic flood model as an alternative to the exponential marginal.

3.3 Theory of Exact Extreme Value Model

Consider a streamflow hydrograph, representing the instantaneous flood peaks at a given station within an interval of time $(0, t]$. Let us consider only those peaks Q_ν, $\nu = 1, 2, \ldots, n$ that exceed the threshold flood level Q_0 as shown in Figure 3.4 and defined

$$\xi_\nu = Q_\nu - Q_0 \qquad\qquad (\xi_\nu > 0) \tag{3.1}$$

According to the nature of flood phenomena, the number of flood exceedances $(\eta(t))$, the magnitude of flood exceedances (ξ_ν) and the time of occurrence of flood exceedances (τ_ν) in a given interval of time $(0, t]$ are random variables.

3.3.1 Distribution Function of the Number of Exceedances

Unlike in the asymptotic extreme value model, the $\eta(t)$ plays an important role in the exact extreme value model because the latter model considers both the $\eta(t)$ and the ξ_ν, simultaneously (Todorovic, 1970). The number of exceedances $\eta(t)$ in $(0, t]$ is defined as

$$\eta(t) = sup\ (\nu) \qquad\qquad (\tau_\nu \leq t) \tag{3.2}$$

By definition, $\eta(t) = 0, 1, 2, \ldots$ for all $t \geq 0$ and $\eta(t) \leq \eta(t + \Delta t)$ for all $t \geq 0$ and $\Delta t > 0$; this means, $\eta(t)$ is a non-decreasing function of t.

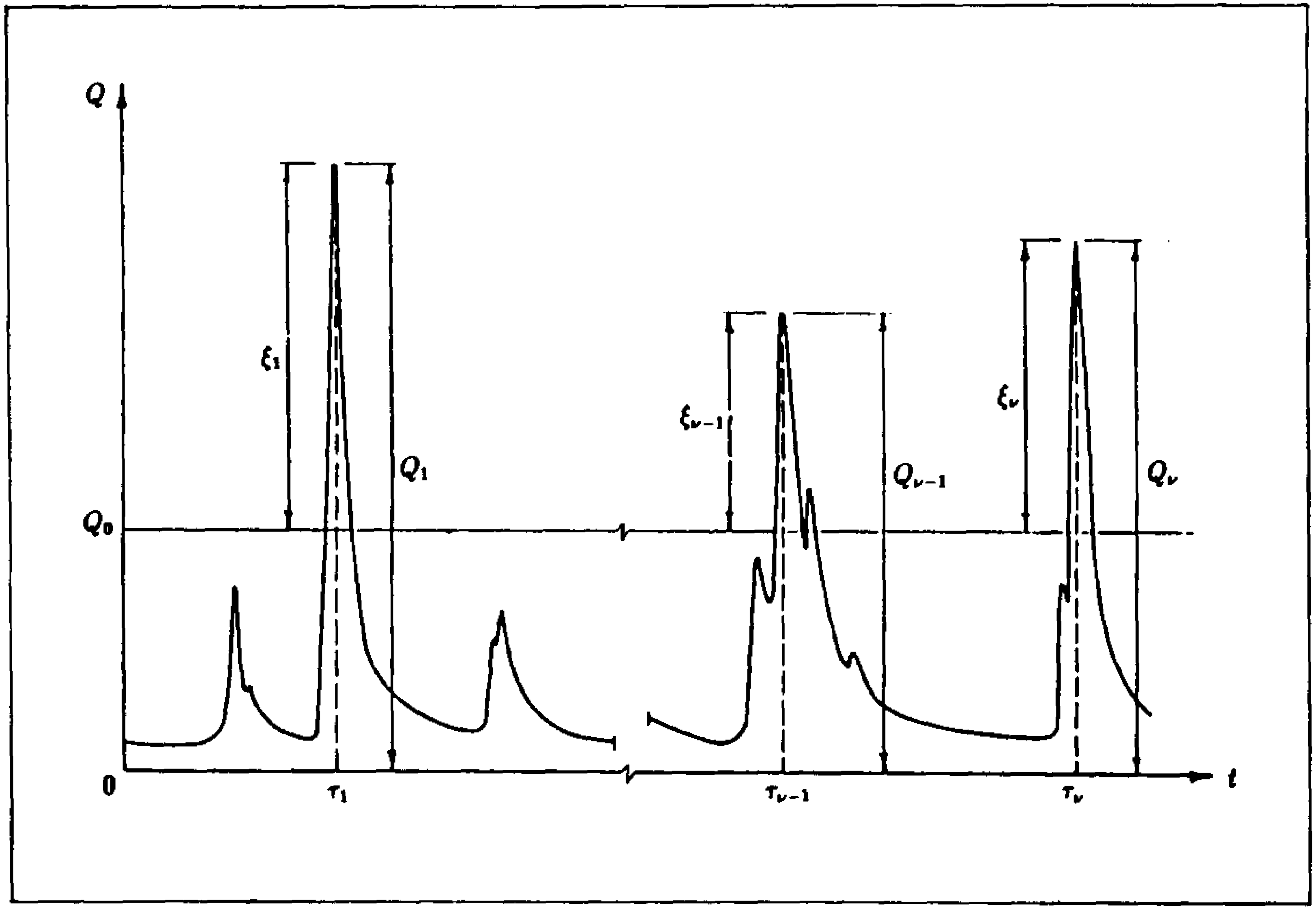


Figure 3.4 A Typical Streamflow Hydrograph of Instantaneous Discharges in a Time Interval $(0,t]$.

Todorovic and Zelenhasic (1970)

In the event that there are exactly ν exceedances occurring in $(0, t]$, defined as (Todorovic, 1970)

$$E_\nu^t = \{\eta(t) = \nu\} \tag{3.3}$$

Following Cox and Miller (1965), for 'very small' time interval $(t, t + \Delta t]$, i.e. in the limit, when $\Delta t \to 0$ only one of the following two events can occur,

: For no exceedance occurrence in $(t, t + \Delta t]$

$$E_0^{t,t+\Delta t} = \{\eta(t + \Delta t) - \eta(t) = 0\} \tag{3.4}$$

: For only one exceedance occurrence in $(t, t + \Delta t]$

$$E_1^{t,t+\Delta t} = \{\eta(t + \Delta t) - \eta(t) = 1\} \tag{3.5}$$

Neglecting the events whose probability of occurrence tends to zero as $\Delta t \to 0$, we can write the following:

$$\begin{aligned} P(E_\nu^{t+\Delta t}) &= P(E_\nu^t)\, P(E_0^{t,t+\Delta t}|E_\nu^t) + P(E_{\nu-1}^t)\, P(E_1^{t,t+\Delta t}|E_{\nu-1}^t) \\ P(E_\nu^{t+\Delta t}) &= P(E_\nu^t)\, \{1 - P(E_1^{t,t+\Delta t}|E_\nu^t)\} + P(E_{\nu-1}^t)\, P(E_1^{t,t+\Delta t}|E_{\nu-1}^t) \end{aligned}$$

Now by rearranging and by dividing with Δt, from above we get

$$\frac{P(E_\nu^{t+\Delta t}) - P(E_\nu^t)}{\Delta t} = \frac{P(E_{\nu-1}^t)\, P(E_1^{t,t+\Delta t}|E_{\nu-1}^t)}{\Delta t} - \frac{P(E_\nu^t)\, P(E_1^{t,t+\Delta t}|E_\nu^t)}{\Delta t}$$

As $\Delta t \to 0$, by letting, $\lambda_\nu^t = \lim_{\Delta t \to 0} \frac{P(E_1^{t,t+\Delta t}|E_\nu^t)}{\Delta t}$ we have the following:

For $\nu = 0$,

$$\frac{dP(E_0^t)}{dt} = -\lambda_0(t)\, P(E_0^t) \tag{3.6}$$

For $\nu = 1, 2, \ldots,$

$$\frac{dP(E_\nu^t)}{dt} = \lambda_{\nu-1}(t)\ P(E_{\nu-1}^t) - \lambda_\nu(t)\ P(E_\nu^t) \tag{3.7}$$

The solution of the above system of differential equations is given by Todorovic and Zelenhasic (1970) as follows:

$$P(E_0^t) = exp\{-\int_0^t \lambda_0(s)ds\} \tag{3.8}$$

$$\begin{aligned} P(E_\nu^t) &= exp\{-\int_0^t \lambda_\nu(s)ds\} \times \int_0^t \lambda_{\nu-1}(s)ds \times \\ & exp\left[\int_0^{t_1}\{\lambda_\nu(s) - \lambda_{\nu-1}(s)\}ds\right] \times \int_0^{t_1} \cdots \int_0^{t_{\nu-1}} \lambda_0(t_\nu) \times \\ & exp\left[\int_0^{t_\nu}\{\lambda_1(s) - \lambda_0(s)\}ds\right]\ dt_\nu\ dt_{\nu-1} \ldots\ldots dt_1 \end{aligned} \tag{3.9}$$

Since, a simple expression for each $P(E_\nu^t)$ in terms of $\lambda_\nu(t)$ is not possible, Todorovic and Yevjevich (1969) have proposed several special cases. In the case of flood analysis, under the hypothesis that $\lambda_\nu(t) = \lambda(t)$ for any ν, Eqn. (3.9) takes the following form (Zelenhasic, 1970):

$$P(E_\nu^t) = \frac{\{\int_0^t \lambda(s)ds\}^\nu}{\nu!}\ exp\{-\int_0^t \lambda(s)ds\} \tag{3.10}$$

where $\lambda(t)$ = mean rate of occurrence of exceedances at time t.

Let $\Lambda(t)$ be the expected value of $\eta(t)$; then by definition we have

$$\Lambda(t) = E[\eta(t)] = \sum_{\nu=1}^{\infty} \nu\ P(E_\nu^t) \tag{3.11}$$

Substituting Eqn. (3.10) into Eqn. (3.11) we get

$$\begin{aligned} \Lambda(t) &= \sum_{\nu=0}^{\infty} \nu\,\frac{\{\int_0^t \lambda(s)ds\}^\nu}{\nu!}\ exp\{-\int_0^t \lambda(s)ds\} \\ &= \sum_{\nu=0}^{\infty} \frac{\{\int_0^t \lambda(s)ds\}^\nu}{(\nu-1)!}\ exp\{-\int_0^t \lambda(s)ds\} \\ &= \int_0^t \lambda(s)ds\ exp\{\int_0^t \lambda(s)ds\}\ exp\{-\int_0^t \lambda(s)ds\} \end{aligned}$$

$$\Lambda(t) = \int_0^t \lambda(s)ds \tag{3.12}$$

Finally, by substituting Eqn (3.12) into Eqn. (3.10) we have

$$P\{\eta(t) = \nu\} = P(E_\nu^t) = \frac{\{\Lambda(t)\}^\nu}{\nu!}\, exp\{-\Lambda(t)\} \tag{3.13}$$

which describes a time dependent Poisson process, where

$\Lambda(t)$ = mean number of exceedances in a specific time interval $(0, t]$.

The function $\lambda(t)$, the mean rate of occurrence at time t, may be determined by more than one flood generating climatic mechanism occurring between time interval $(0, t]$. However, for an individual sub-population with hydroclimatically similar flood events, the $\lambda(t)$ can be considered as a constant (λ) between the time interval $(0, t]$. Therefore, Eqn. (3.13) becomes a time independent Poisson process for the k^{th} sub-population, with λ_k as the Poisson parameter (k = 1, 2, ..., N_s). By substituting $\Lambda(t) = \lambda_k.t$ into Eqn. (3.13) we get

$$P\{\eta_k(t) = \nu_k\} = P(E_{\nu_k}^t) = \frac{\{\lambda_k.t\}^{\nu_k}}{\nu_k!}\, exp(-\lambda_k.t) \tag{3.14}$$

3.3.2 Distribution Function of the Largest Exceedance

In the case of flood analysis, the random variable of the largest exceedance $(\chi(t))$, among the set of all exceedances (ξ_ν) within a specific time interval $(0, t]$ plays an important role.

Following Todorovic (1970), $\chi(t)$ is defined as

$$\chi(t) = sup\ (\xi_\nu) \qquad\qquad (\tau_\nu \leq t) \tag{3.15}$$

By definition, $\chi(t) \leq \chi(t + \Delta t)$ for all $t \geq 0$ and $\Delta t > 0$; this means that $\chi(t)$ is a non-decreasing stochastic process.

Denote by $\phi_t(x)$ the distribution function of $\chi(t)$, i.e.

$$\phi_t(x) = P\{\chi(t) \le x\}, \qquad (t \ge 0,\ \ x \ge 0) \tag{3.16}$$

Todorovic (1970) derived an expression for $\phi_t(x)$, on the basis of mathematical expectation of the conditional probability, $P\{\chi(t) \le x | \eta(t)\}$ as

$$\begin{aligned}
\phi_t(x) &= E[P\{\chi(t) \le x|\eta(t)\}] \\
&= E[P\{sup\,(\xi_\nu) \le x|\eta(t)\}] \qquad (\tau_\nu \le t) \\
&= \sum_{n=0}^{\infty} P[\{sup\,(\xi_\nu) \le x\} \cap E_n^t] \qquad (\tau_\nu \le t,\ 0 \le \nu \le n) \\
\phi_t(x) &= \sum_{n=0}^{\infty} P\{\bigcap_{\nu=0}^{n}(\xi_\nu \le x) \cap E_n^t\}
\end{aligned} \tag{3.17}$$

Eqn. (3.17) describes the most general form of Todorovic's exact extreme value stochastic flood model for distribution function of the largest exceedances $[\phi_t(x) = P\{\chi(t) \le x\}]$ in a specific time period $(0,\ t]$.

3.4 Various Forms of the Stochastic Model

The most general form of the model described by Eqn. (3.17) is difficult to solve directly unless one determines the probability $P\{\bigcap_{\nu=0}^{n}(\xi_\nu \le x) \cap E_n^t\}$. Therefore, it is necessary to consider a particular form of the Eqn. (3.17) in which the ξ_ν ($\nu = 1, 2, \ldots, n$) occurring in $(0, t]$ are i.i.d. and the random sequences ξ_ν and τ_ν are stochastically independent for all ν. Under these assumptions, Eqn. (3.17) simplifies to

$$\begin{aligned}
\phi_t(x) &= \sum_{n=0}^{\infty} \{P(\xi_\nu \le x)\}^n\ P(E_n^t) \\
\phi_t(x) &= \sum_{n=0}^{\infty} \{F(x)\}^n\ P(E_n^t)
\end{aligned} \tag{3.18}$$

where $F(x) = P(\xi_\nu \leq x)$, marginal distribution function of ξ_ν in $(0, t]$.

According to Ashkar and Rousselle (1987), the stochastic independency between extreme events can be considered as a valid assumption when the number of exceedances describes a Poissonian process as in Eqn. (3.14). The other important assumption that flood exceedances are identically distributed can be met by applying the model to each hydroclimatically separated homogeneous sup-population. Therefore, writing Eqn. (3.18) for the k^{th} sub-population with Poissonian assumption described by Eqn. (3.14) and by letting, $t = 1$ year, we obtain

$$\begin{aligned}
\phi_k(x) &= \sum_{n=0}^{\infty} \frac{(\lambda_k.1)^n}{n!}\, exp(-\lambda_k)\, \{F_k(x)\}^n \\
&= exp(-\lambda_k) \sum_{n=0}^{\infty} \frac{\{\lambda_k\, F_k(x)\}^n}{n!} \\
&= exp(-\lambda_k)\, exp\{\lambda_k\, F_k(x)\} \\
\phi_k(x) &= exp[-\lambda_k\{1 - F_k(x)\}] \qquad (3.19)
\end{aligned}$$

where

$\phi_k(x)$ = distribution function of the largest exceedance of the k^{th} sub-population,

$F_k(x)$ = marginal distribution function of the k^{th} sub-population, and

λ_k = Poisson distribution parameter of the k^{th} sub-population ($k = 1, 2, \ldots, N_s$).

For N_s independent flood generating processes (i.e. hydroclimatically distinct sub-populations), the overall annual distribution of the largest exceedances can be written as

$$\begin{aligned}
\phi(x) &= exp[-\lambda_1\{1 - F_1(x)\}]\, exp[-\lambda_2\{1 - F_2(x)\}] ... exp[-\lambda_{N_s}\{1 - F_{N_s}(x)\}] \\
\phi(x) &= exp[-\sum_{k=1}^{N_s} \lambda_k\{1 - F_k(x)\}] \qquad (3.20)
\end{aligned}$$

where

$F_i(.) \neq F_j(.)$ for all i and j $(i, j = 1, 2, \ldots, N_s)$,

$\phi(x)$ = distribution function of the largest annual flood exceedance, and

N_s = number of hydroclimatically distinct sup-populations.

3.4.1 Marginal Distribution of the Model

Exponential Marginal:

The exponential distribution is the most widely used marginal distribution in existing exact extreme value stochastic flood models as a common distribution function of all exceedances ξ_ν, $\nu = 0, 1, 2, \ldots$, in a given interval of time $(0, t]$. Consider all exceedances of the k^{th} sub-population are i.i.d. with a common distribution function in an exponential form. The pdf and cdf of exponentially distributed random variable $\xi_{\nu k}$ can be expressed, respectively, as

$$f_k(x) = \beta_k \, exp(-\beta_k x) \qquad (x \geq 0, \ \beta_k > 0) \tag{3.21}$$

$$F_k(x) = P(\xi_{\nu k} \leq x) = 1 - exp(-\beta_k x) \tag{3.22}$$

Substituting Eqn. (3.22) into Eqn. (3.20) we get the existing form of the exact extreme value stochastic model as (exponential model),

$$\phi(x) = exp\{-\sum_{k=1}^{N} \lambda_k \, e^{-\beta_k x}\} \tag{3.23}$$

where $\beta_k^{-1} = E[\xi_{\nu k}]$ = mean of the exceedances of the k^{th} sub-population (k = 1, 2, ..., N_s).

As the exponential distribution has coefficient of variation equal to unity and its only parameter β_k absorbs a lesser degree of freedom when fitted to a given sample, the hypothesis that the sample is exponentially distributed is important and must be met for this model. Therefore, if necessary it may be required to raise the Poisson admissible threshold level to meet the exponential hypothesis. However, raising of the threshold level censors out the variability of the data we are trying to model and also reduces the sample size and thereby increases the uncertainty in parameter estimation. To avoid this, it may be advantageous in some situations to maintain the Poisson admissible threshold level and search for alternative marginal distributions.

Weibull Marginal:

When introducing a feasible marginal distribution to the model given by Eqn. (3.20), it may be necessary to examine the following properties of the selected marginal distribution: *a*) parismony with respect to parameters, *b*) availability of stable and efficient parameter estimation technique, *c*) robustness of asymptotic performances, *d*) existence of closed form inverse function of cdf such that estimation by PWM is possible, and *e*) be a member of exponential family. In view of these properties, the two parameter Weibull distribution was considered as the marginal in the proposed model (Weibull model).

The Weibull contains the exponential as a special case (a=1) as shown in Figure 3.5. As the Weibull marginal contains an additional parameter compared to the exponential counterpart, an additional source of sampling error is expected in the

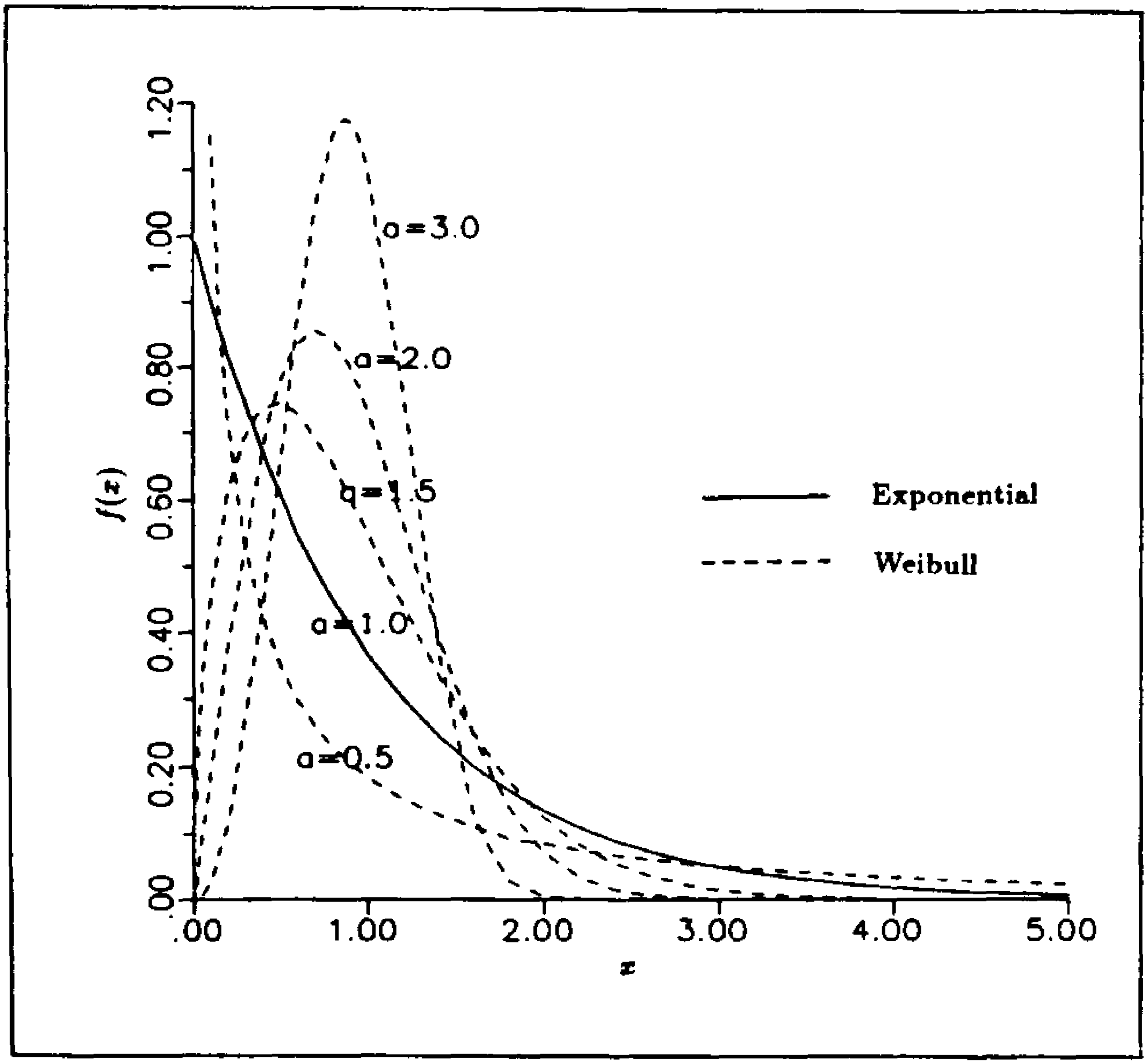


Figure 3.5 Shapes of Weibull Distribution for Different Values of Parameter a: a=1.0, Represents Exponential Distribution (Singh, et al., 1990).

Weibull model. However, the Weibull distribution can be used to describe flood samples having coefficients of variation (CV) in a much wider range as shown by the following (Singh et al., 1990):

$$\begin{aligned} \mu &= b\,\Gamma(1+1/a) \\ \sigma^2 &= b^2\,\{\Gamma(1+2/a) - \Gamma^2(1+1/a)\} \\ CV &= \frac{\{\Gamma(1+2/a) - \Gamma^2(1+1/a)\}^{1/2}}{\Gamma(1+1/a)} \end{aligned} \tag{3.24}$$

Thus, when using the Weibull distribution in fitting a given flood sample with CV defined by Eqn. (3.24), the lower Poisson accepted threshold level may sometimes be used to define the PDS. Hence, this Poisson accepted threshold level in the Weibull model may define a larger sample size when compared to the exponentially accepted threshold level, thereby reducing the sampling error. Therefore, in this study, as one of the important goals, the sampling error of the Weibull model relative to the exponential model is examined for various population conditions expected in the real world observed flood sequences. Thus, the conditions under which the Weibull model would be preferred can be determined.

The pdf and cdf of Weibull distributed random variable $\xi_{\nu k}$ can be expressed, respectively as (Singh et al., 1990)

$$f_k(x) = \frac{a_k}{b_k}\left(\frac{x}{b_k}\right)^{a_k-1} exp\{-(x/b_k)^{a_k}\} \qquad (x \geq 0,\ \ a_k,\ b_k > 0) \tag{3.25}$$

$$F_k(x) = P(\xi_{\nu k} \leq x) = 1 - exp\{-(x/b_k)^{a_k}\} \tag{3.26}$$

Substituting Eqn. (3.26) into Eqn. (3.20) the proposed model takes the following form:

$$\phi(x) = exp\{-\sum_{k=1}^{N} \lambda_k e^{-(x/b_k)^{a_k}}\} \tag{3.27}$$

where

a_k = shape parameter of the Weibull distribution for the k^{th} sub-population, and

b_k = scale parameter of the Weibull distribution for the k^{th} sub-population.

Note: when $a_k = 1$, Eqn. (3.27) simplifies to Eqn. (3.23).

3.4.2 Mixed Populations in the Model

In minimizing the number of parameters in the model, it is necessary to determine the statistical similarities of flood groups in a given flood sequence produced by different climatic mechanisms. Those flood groups which are found to be statistically similar are combined into one sub-population, thereby minimizing the number of parameters in the model.

When the flood sequence is left with just one such sub-population (i.e. k=1) Eqn. (3.23) and (3.27) represent the 'simple' formulations of the existing and proposed models, respectively, as

$$\phi(x) = exp(-\lambda e^{-\beta x}) \tag{3.28}$$

$$\phi(x) = exp\{-\lambda e^{-(x/b)^{a}}\} \tag{3.29}$$

Otherwise, when the flood sequence is left with more than one sub-population (i.e. say, k=2) the Eqn. (3.23) and (3.27) represent the 'compound' formulations of the existing and proposed models, respectively, as

$$\phi(x) = exp(-\lambda_1 e^{-\beta_1 x} - \lambda_2 e^{-\beta_2 x}) \tag{3.30}$$

$$\phi(x) = exp\{-\lambda_1 e^{-(x/b_1)^{a_1}} - \lambda_2 e^{-(x/b_2)^{a_2}}\} \tag{3.31}$$

In this study, to preserve the statistical attractiveness of the compound model by having a minimum numbers of parameters, at most two distinct sub-populations ($k=2$) are considered for mixed flood populations. The statistical properties of both simple and compound formulations were analyzed under both the exponential and Weibull marginal assumptions in terms of predictive and descriptive performances.

3.5 Estimation of Model Parameters

The Poisson parameter λ which describes the random variable $\eta(t)$ is estimated as the mean rate of occurrence of flood exceedances during a time interval $(0,\ t]$,

$$\hat{\lambda} = \frac{n}{t} \tag{3.32}$$

where

$\hat{\lambda}$ = estimate of the Poisson parameter λ,

n = number of flood exceedances during time interval $(0, t]$, and

t = a period of one water year (October through September).

The marginal distribution which describes the random variable ξ_ν, if considered as exponentially distributed, has parameter β estimated by MLE as

$$\hat{\beta} = 1/\hat{\mu} = 1/(1/n)\sum_{i=1}^{n} x_i \tag{3.33}$$

where

$\hat{\beta}$ = estimate of the exponential parameter β, and

$\hat{\mu}$ = sample estimate of the population mean μ of random variable ξ_ν.

However, when the Weibull distribution is considered as the marginal distribution, the statistical characteristics such as $BIAS$, SE and $RMSE$ of estimated parameters ($\hat{a}$ and $\hat{b}$) and flood quantiles ($\hat{Q}_T$) are influenced by the selected method of parameter estimation. Therefore, following the study by Singh et al. (1990), the Weibull parameters were estimated using both the POME and PWM estimators.

3.5.1 Principle of Maximum Entropy

Shannon (1948a, 1948b) defined the Shannon entropy functional (SEF) of probability distribution function $f(x)$, in describing the random variable ξ_ν as

$$I[f] = -\int_{-\infty}^{\infty} f(x)\, ln f(x) dx \qquad (\int_{-\infty}^{\infty} f(x) dx = 1) \tag{3.34}$$

Jaynes (1961, 1982) introduced the POME to estimate the least biased $f(x)$ by maximizing SEF $I[f]$, subject to the given information or constraints. Singh (1987) derived the following constraints in estimating the parameters of the Weibull distribution as:

$$\int_0^{\infty} f(x) dx = 1 \tag{3.35}$$

$$\int_0^{\infty} ln\ x\ f(x) dx = E[ln\ \xi_\nu] \tag{3.36}$$

$$\int_0^{\infty} x^a\ f(x) dx = E[\xi_\nu^a] = b^a \tag{3.37}$$

Writing the entropy representation of least biased $f(x)$, consistent with Eqns. (3.35), (3.36) and (3.37) leads to the following form (Singh, 1987),

$$f(x) = exp(-\lambda_0 - \lambda_f\ ln\ x - \lambda_s\ x^a) \tag{3.38}$$

where λ_0, λ_f, and λ_s are Lagrange multipliers, which can be determined from Eqns. (3.36), (3.37) and (3.38) along with the normalized condition, $\int_0^\infty f(x)dx = 1$. Substituting Eqn. (3.38) into Eqn. (3.35) we get

$$\int_0^\infty exp(-\lambda_0 - \lambda_f \, ln\, x - \lambda_s \, x^a)dx = 1$$

$$\int_0^\infty exp\{(ln\, x)^{-\lambda_f} - \lambda_s \, x^a\}dx = exp(\lambda_0) \tag{3.39}$$

By letting $y = \lambda_s \, x^a$ in Eqn. (3.39), then by integrating and simplifying yields the zeroth Lagrange multiplier λ_0 in terms of constraints as (Singh, 1987),

$$\lambda_0 = ln\, \Gamma\left(\frac{1-\lambda_f}{a}\right) - ln\, a - \left(\frac{1-\lambda_f}{a}\right) \, ln\, \lambda_s \tag{3.40}$$

The other Lagrange multipliers λ_f and λ_s in terms of constraints are obtained in the following manner. From Eqn. (3.39) we have

$$\lambda_0 = ln \int_0^\infty exp(-\lambda_f \, ln\, x - \lambda_s \, x^a)dx \tag{3.41}$$

Taking the partial derivatives of Eqn. (3.41) w.r.t. λ_f and λ_s, then simplifying and equating them to Eqns. (3.36) and (3.37) respectively, we get the following:

$$\frac{\partial \lambda_0}{\partial \lambda_f} = -\int_0^\infty ln\, x \, f(x)dx = -E[ln\, \xi_\nu] \tag{3.42}$$

$$\frac{\partial \lambda_0}{\partial \lambda_s} = -\int_0^\infty x^a \, f(x)dx = -E[\xi_\nu^a] = -b^a \tag{3.43}$$

Also, taking the partial derivatives of Eqn. (3.40) w.r.t. λ_f and λ_s, respectively, yields the following:

$$\frac{\partial \lambda_0}{\partial \lambda_f} = \frac{\partial}{\partial \lambda_f}\left\{ln\, \Gamma\left(\frac{1-\lambda_f}{a}\right)\right\} + \frac{ln\, \lambda_s}{a} \tag{3.44}$$

$$\frac{\partial \lambda_0}{\partial \lambda_s} = -\left(\frac{1-\lambda_f}{a}\right) \frac{1}{\lambda_s} \tag{3.45}$$

Therefore, by equating Eqn. (3.42) to (3.44) and Eqn. (3.43) to (3.45) we get the Lagrange multipliers λ_f and λ_s in terms of constraints as follows (Singh, 1987):

$$\frac{\partial}{\partial \lambda_f}\left\{ln\,\Gamma\left(\frac{1-\lambda_f}{a}\right)\right\} + \frac{ln\,\lambda_s}{a} = -E[ln\,\xi_\nu] \tag{3.46}$$

$$\left(\frac{1-\lambda_f}{a}\right)\frac{1}{\lambda_s} = E[\xi_\nu^a] = b^a \tag{3.47}$$

Substituting Eqn. (3.40) into Eqn. (3.38) and simplifying we get

$$f(x) = \frac{1}{\Gamma\left(\frac{1-\lambda_f}{a}\right)}\, a\, \lambda_s^{(1-\lambda_f)/a}\, x^{-\lambda_f}\, exp(-\lambda_s\, x^a) \tag{3.48}$$

Eqn. (3.48) reduces to Eqn. (3.25) after omitting the k^{th} subscript. The Lagrange multipliers λ_f and λ_s relate to the parameters a and b as (Singh, 1987)

$$\lambda_f = 1 - a \tag{3.49}$$

$$\lambda_s = \frac{1}{b^a} \tag{3.50}$$

The relationships of Lagrange parameters (λ_f and λ_s) to the known constraints given by Eqns. (3.46) and (3.47) and Weibull parameters (a and b) given by Eqns. (3.49) and (3.50) can be used to relate the parameters directly to the known constraints as follows (Singh, 1987):

$$b^a = E[\xi_\nu] \tag{3.51}$$

$$\frac{1}{a}\,\psi(1) + ln\,b = E[ln\,\xi_\nu] \tag{3.52}$$

Therefore, the entropy estimates of Weibull parameters, which describe the marginal distribution of random variable ξ_ν can be estimated as follows:

$$\hat{b}^{\hat{a}} = 1/(1/n)\sum_{i=1}^{n} x_i^{\hat{a}} \tag{3.53}$$

$$\frac{1}{\hat{a}}\,\psi(1) + ln\,\hat{b} = 1/(1/n)\sum_{i=1}^{n} ln\, x_i \tag{3.54}$$

where

$\psi()$ = psi function, and

$\hat{a}$ and $\hat{b}$ = estimates of Weibull parameters a and b, respectively.

3.5.2 Probability Weighted Moments

Greenwood et al. (1979) defined the PWM's of a random variable ξ_ν with cdf $F(x) = P(\xi_\nu \le x)$ as,

$$M_{r,s,t} = E[\{\xi_\nu\}^r\,\{F(x)\}^s\,\{1 - F(x)\}^t] \tag{3.55}$$

where r, s, and t are real non-negative numbers.

When the cdf has a closed form inverse function $x(F)$, Eqn. (3.55) takes the following form (Greenwood et al., 1979):

$$M_{r,s,t} = \int_0^1 \{x(F)\}\,\{F\}^s\,\{1 - F\}^t\,dF \tag{3.56}$$

In deriving analytically simple relations between the PWM's and the distribution parameters, Greenwood et al. (1979) considered the following form of Eqn. (3.56) ($r = 1$, $s = 0$ and $t = 1, 2, \ldots, t$):

$$M_{(t)} = M_{1,0,t} = \int_0^1 \{x(F)\}\,\{1 - F\}^t\,dF \tag{3.57}$$

By substituting the inverse function $x(F)$ of Eqn. (3.26) without the k^{th} subscript into Eqn. (3.57), the PWM's of Weibull distribution can be determined as follows:

$$M_{(t)} = \int_0^1 b\{-ln(1-F)\}^{1/a}\,\{1-F\}^t\,dF \tag{3.58}$$

By substituting Eqn. (3.26) without the k^{th} subscript into Eqn. (3.58) and simplifying we get

$$\begin{aligned} M_{(t)} &= b\int_0^\infty \left(\frac{x}{b}\right)\,[exp\{-(x/b)^a\}]^t\,\frac{a}{b}\,\left(\frac{x}{b}\right)^{a-1} exp\{-(x/b)^a\}\,dx \\ M_{(t)} &= a\int_0^\infty \left(\frac{x}{b}\right)^a\,exp\{-(1+t)(x/b)^a\}\,dx \end{aligned} \tag{3.59}$$

By letting $u = (x/b)^a$ in Eqn. (3.59) and simplifying yields the following:

$$M_{(t)} = b\int_0^\infty u^{1/a}\,exp\{-(1+t)u\}\,du \tag{3.60}$$

Again by letting $z = (1+t)u$ in Eqn. (3.60) we have the PWM's of Weibull distribution as

$$\begin{aligned} M_{(t)} &= \frac{b}{(1+t)^{1/a}}\int_0^\infty z^{1/a}\,exp(-z)\,\frac{1}{(1+t)}\,dz \\ M_{(t)} &= \frac{b}{(1+t)^{(1+1/a)}}\int_0^\infty z^{(1/a+1-1)}\,exp(-z)\,dz \\ M_{(t)} &= \frac{b\,\Gamma(1+1/a)}{(1+t)^{(1+1/a)}} \end{aligned} \tag{3.61}$$

In the case of Weibull distribution, the first two PWM's ($M_{(0)}$ and $M_{(1)}$) in Eqn. (3.61) are sufficient to estimate the parameters a and b. Thus, the PWM estimators of a and b were found as (Greenwood et al., 1979):

$$\hat{a} = \frac{ln\,(2)}{ln\,\{M_{(0)}/2\,M_{(1)}\}} \tag{3.62}$$

$$\hat{b} = \frac{M(0)}{\Gamma\{ln\,(M_{(0)}/M_{(1)})/ln\,(2)\}} \tag{3.63}$$

Chapter 4

Methodology

4.1 Introduction

Flood frequency analysis involves selecting a flood-like probability distribution and estimating its parameters and quantiles using a robust estimator. In selecting a flood-like probability distribution, both simple and compound forms of the proposed Weibull model are considered as an alternative to both forms of the exponential model. Hence, this chapter will be concentrated on evaluation of both the predictive and descriptive performances of the above exact extreme value stochastic flood models. In achieving this, the following major tasks will be performed.

1. Comparison of asymptotic predictive performance of the proposed Weibull model to that of the exponential model by Monte-Carlo simulation,

2. Evaluation of asymptotic predictive performances of POME and PWM estimators for the Weibull model for various population conditions,

3. Investigation of the effect of hydroclimatic separation of annually nonhomogeneous populations into homogeneous sub-populations, using the properties of predictive performances of both the exponential and Weibull models,

4. Comparison of the descriptive statistical performance of the Weibull model to that of the exponential model using observed flood sequences, and

5. Investigation of the descriptive statistical performances of the effect of hydroclimatic separation of annually nonhomogeneous observed flood sequences using both the exponential and Weibull models.

4.2 Performance Indices of Estimators

The procedure for estimating the parameters and quantiles for a given probability model is to use an observed random sample. Thus, $\hat{\theta}$ an estimate for the parameter or quantile θ is a function of the observations or random variables. Since, $\hat{\theta}$ is itself a random variable its performance has to be evaluated statistically for various population conditions. Define the following relative performance indices of $\hat{\theta}$ as;

$$BIAS(\hat{\theta}) = \frac{E(\hat{\theta}) - \theta}{\theta} \tag{4.1}$$

$$SE(\hat{\theta}) = \frac{\{E[\hat{\theta} - E(\hat{\theta})]^2\}^{1/2}}{\theta} = \frac{\sigma(\hat{\theta})}{\theta} \tag{4.2}$$

$$RMSE(\hat{\theta}) = \frac{\{E(\hat{\theta} - \theta)^2\}^{1/2}}{\theta} = \{BIAS(\hat{\theta})^2 + \frac{N-1}{N} SE(\hat{\theta})^2\}^{1/2} \tag{4.3}$$

where

N = number of random samples generated for each sample of size n,

$E(\hat{\theta})$ = statistical expectation of estimate of θ,

$\sigma(\hat{\theta})$ = standard deviation of estimate of θ,

$BIAS(\hat{\theta})$ = relative bias of estimate of θ,

$SE(\hat{\theta})$ = relative standard error of estimate of θ, and

$RMSE(\hat{\theta})$ = relative root mean square error of estimate of θ.

The above performance indices can be used to define a more robust flood estimator for $\hat{\theta}$ as one that is 'resistant' and 'efficient', over a wide range of anticipated population conditions (Kuczera, 1982). The estimator is said to be resistant, when the average $RMSE(\hat{\theta})$ is minimum and is said to be efficient, when the $BIAS(\hat{\theta})$ and the $SE(\hat{\theta})$ are small as that of any other estimator over the wide range of all population variations. In this study, the above performance indices were computed from Monte-Carlo based simulated data.

4.3 Analysis of Simulated Data

In computing the performance indices, the advantage of using simulated data is that the data are error-free and can be generated for any population conditions encountered in practice, according to a given probability distribution. Therefore, the Monte-Carlo based sampling experiments were used to examine a flood-like probability model (task 1) and to choose the most robust estimator (task 2) for the selected probabilistic model. In addition, these sampling experiments were also used to find out the effect of hydroclimatic separation of a nonhomogeneous population into homogeneous sub-populations (task 3).

4.3.1 Population Parameters

Task 1: Exponential Vs Weibull Marginal

In assessing the task 1, the comparative performance of the Weibull model relative to the exponential model, the Monte-Carlo based random samples were generated according to the annually homogeneous population cases listed in Table 4.1 and Table 4.2.

Table 4.1. Exponential Model - Homogeneous Populations

Case No.	λ	μ
1	1.0	1.0
2	2.0	1.0
3	3.0	1.0

Table 4.2. Weibull Model - Homogeneous Populations

Case No.	λ	μ	CV
1	1.0	1.0	0.5
2	2.0	1.0	0.5
3	3.0	1.0	0.5
4	1.0	1.0	1.0
5	2.0	1.0	1.0
6	3.0	1.0	1.0
7	1.0	1.0	1.5
8	2.0	1.0	1.5
9	3.0	1.0	1.5
10	1.0	1.0	3.0
11	2.0	1.0	3.0
12	3.0	1.0	3.0

In the case of the Weibull model, the CV was selected to cover a wide range of population conditions expected in the observed flood series of Louisiana as seen from Figure 4.1. It also shows that the estimated values of the Weibull shape parameter ($\hat{a}$) are always falling below the line of $a = 1$ for those observed flood series which were used in the final analysis of this study. This implies that for any selected series, the shape of the Weibull distribution should be an exponential per se, where $a \leq 1.0$ (see Figure 3.5) in order to describe those observed flood series. Furthermore, the Weibull distribution can be used as a feasible 'flood' probability distribution only when $a \leq 1.0$, in which there exist a zero lower bound and no upper bound. Therefore, for various values of CV's the performance of the quantile estimates of the Weibull model could be compared to that of the exponential model.

In the case of the exponential model, the Poisson mean (λ) was allowed to vary from 1.0 to 3.0, while the mean of the magnitude of exceedances (μ) was held constant at 1.0. In the case of the Weibull model, for a given value of CV of exceedances (0.5, 1.0, 1,5 or 3.0), λ was allowed to vary from 1.0 to 3.0, while μ was held constant at 1.0. The random samples generated according to the above population conditions could be used to evaluate the relative performance of estimates of flood quantiles of the Weibull model (Eqn. 3.29) to the exponential model (Eqn. 3.28) for varying population CV and threshold level as exhibited by the number of exceedances per year.

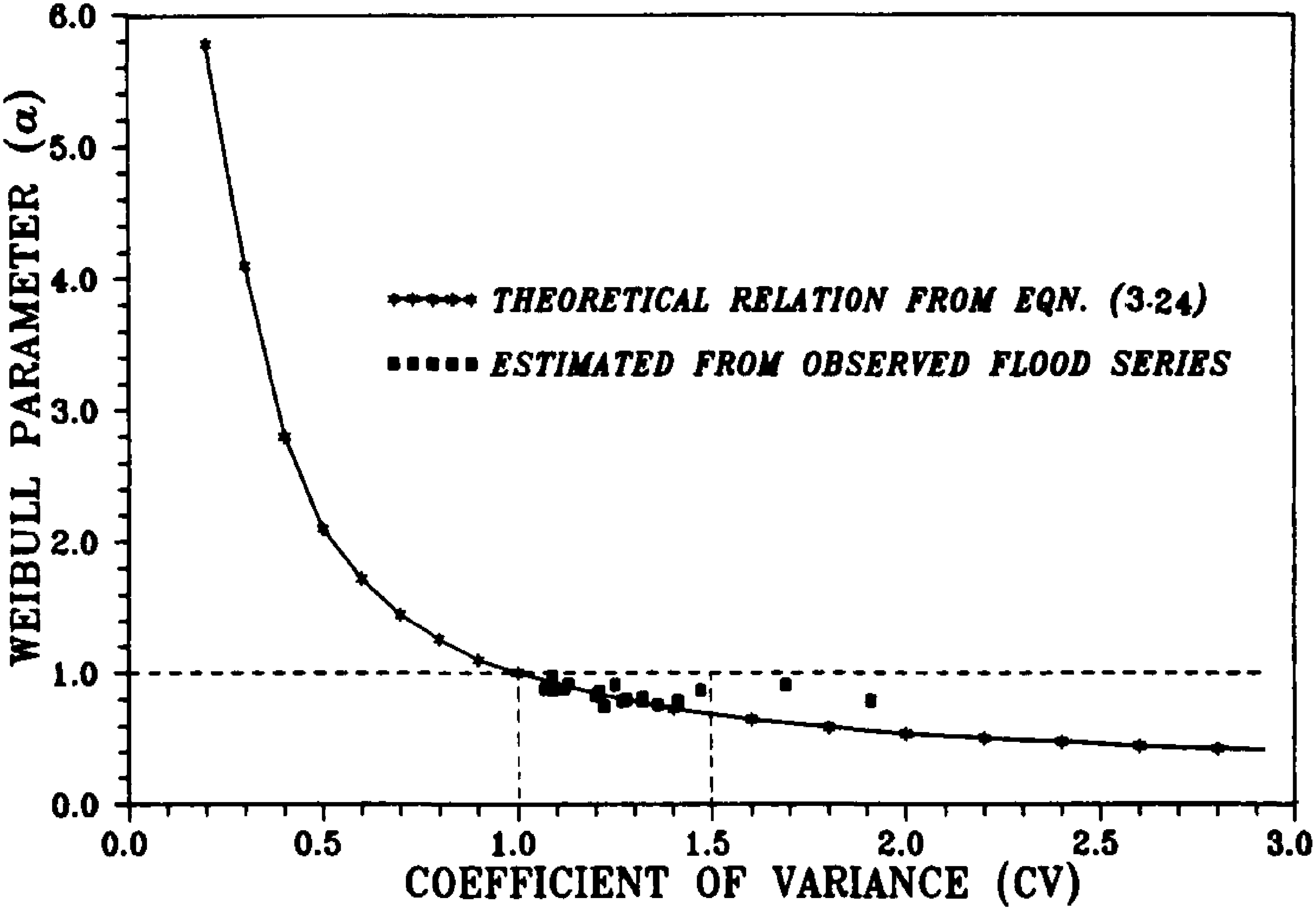


Figure 4.1 Weibull Shape Parameter Vs Population Coefficient of Variance

Task 2: POME Vs PWM Estimator

In assessing the task 2, following the results of Singh et al. (1990), the asymptotic properties of the POME and PWM estimators for the Weibull model were evaluated based on Monte-Carlo random samples generated according to annually homogeneous Weibull population cases as shown in Table 4.2. In these cases, population CV values were allowed to vary from 0.5 to 3.0 so that the performance of estimates of model parameters and quantiles could be compared.

Task 3: Mixed Vs Separated Sub-Populations

In analyzing the task 3, the evaluation of the effect of hydroclimatic separation of annually nonhomogeneous population into homogeneous sub-populations, the Monte-Carlo based random samples were generated for both exponential and Weibull models as listed in Table 4.3 and 4.4. For mixed sub-populations, the simulated data of sub-population 1 and 2 were first combined before estimating the parameters of the simple forms of the exponential (Eqn. 3.28) and Weibull (Eqn. 3.29) models. But in the case of separated sup-populations, the simulated data of each sub-population were considered separately in estimating the parameters of the compound forms of the exponential (Eqn. 3.30) and Weibull (Eqn. 3.31) models.

The estimates of flood quantiles of both formulations of the exponential model could be compared to evaluate the effect of hydroclimatic separation of mixed populations in terms of the means. Conversely, the estimates of flood quantiles of both formulations of the Weibull model could be compared to evaluate the effect of hydroclimatic separation in terms of the variances.

Table 4.3. Exponential Model - Nonhomogeneous Populations

Case	Sub-Population 1		Sub-Population 2	
No.	λ_1	μ_1	λ_2	μ_2
1	0.7	1.0	0.3	1.5
2	1.3	1.0	0.7	1.5
3	2.0	1.0	1.0	1.5
4	0.7	1.0	0.3	2.0
5	1.3	1.0	0.7	2.0
6	2.0	1.0	1.0	2.0
7	0.7	1.0	0.3	3.0
8	1.3	1.0	0.7	3.0
9	2.0	1.0	1.0	3.0

Table 4.4. Weibull Model - Nonhomogeneous Populations

Case	Sub-Population 1			Sub-Population 2		
No.	λ_1	μ_1	CV_1	λ_2	μ_2	CV_2
1	1.0	1.0	0.5	0.5	1.5	1.0
2	1.3	1.0	0.5	0.7	1.5	1.0
3	2.0	1.0	0.5	1.0	1.5	1.0
4	1.0	1.0	0.5	0.5	1.5	1.5
5	1.3	1.0	0.5	0.7	1.5	1.5
6	2.0	1.0	0.5	1.0	1.5	1.5
7	1.0	1.0	0.5	0.5	1.5	3.0
8	1.3	1.0	0.5	0.7	1.5	3.0
9	2.0	1.0	0.5	1.0	1.5	3.0

4.3.2 Monte-Carlo Simulation

In estimating the performance indices of $\hat{\theta}$ through Monte-Carlo simulation, the following procedure was adopted for each population case listed in Table 4.1 to Table 4.4:

1. Select a number of years of record n (10, 20, 30, 50, 100, 200, 500 or 1000),

2. (a). For Homogeneous Populations:

   Choose a Poisson parameter λ (1, 2 or 3) and generate n number of Poisson random deviates as $\tilde{\lambda}_1, \tilde{\lambda}_2, \ldots, \tilde{\lambda}_n$,

   (b). For Nonhomogeneous Populations:

   Choose two Poisson parameters λ_1 and λ_2 (s.t. $\lambda_1 + \lambda_2 = 1, 2,$ or 3) and generate two independent random samples each having n number of Poisson deviates as $\tilde{\lambda}_{11}, \tilde{\lambda}_{12}, \ldots, \tilde{\lambda}_{1n}$ and $\tilde{\lambda}_{21}, \tilde{\lambda}_{22}, \ldots, \tilde{\lambda}_{2n}$ for sub-population 1 and 2 respectively,

3. (a). For Homogeneous Populations:

   Generate $\sum_{j=1}^{n} \tilde{\lambda}_j$ number of random deviates for each of the following distribution:

   (i). Exponential with μ as in Table 4.1

   (ii). Weibull with μ, CV as in Table 4.2,

   (b). For Nonhomogeneous Populations:

   Generate two independent random samples with $\sum_{j=1}^{n} \tilde{\lambda}_{1j}$ and $\sum_{j=1}^{n} \tilde{\lambda}_{2j}$ number of random deviates for each of the following distribution:

(i). Exponential with μ_1 and μ_2 as in Table 4.3

(ii). Weibull with μ_1, CV_1 and μ_2, CV_2 as in Table 4.4

4. Repeat step 2 to 3 N times to generate that many number of both exponential and Weibull distributed homogeneous and nonhomogeneous random samples,

5. Estimate $\hat{\theta}_i$ ($i = 1, 2, \ldots, N$) from the chosen estimator for each of the N random sample of size n for all population cases listed, and

6. Compute the $E(\hat{\theta})$ and $\sigma(\hat{\theta})$ for each of the population case as

$$E(\hat{\theta}) \cong 1/N \sum_{i=1}^{N} \hat{\theta}_i \tag{4.4}$$

$$\sigma(\hat{\theta}) \cong \{1/(N-1) \sum_{i=1}^{N} [\hat{\theta}_i - E(\hat{\theta})]^2\}^{1/2} \tag{4.5}$$

In this way, for each population case with number of years of record n, the performance indices defined in Eqn. (4.1) to (4.3) were determined. The results of these performance indices are given in Tables A.1 through A.126 in executing the tasks 1 through 3.

The relative performances of the flood models did not vary with number of random samples (N) generated if more than 1000 samples were used. Therefore, only the results based on 1000 random samples are discussed as approximate values of performance indices of $BIAS(\hat{\theta})$ and $RMSE(\hat{\theta})$ for each n and population case listed. The number of years of record n was selected to cover a wide range of values from 10 to 1000, particularly number of years of record mostly encountered in hydrologic practise such as 10, 20, 30, and 50.

4.4 Analysis of Observed Data

Following the analysis of the predictive properties of selected flood probability models, the descriptive properties of those models were tested on observed data as realistic population conditions. Hence, the observed flood series were used to compare the descriptive properties of the Weibull model to that of the exponential model (task 4) and to analyze the effect of hydroclimatic separation of nonhomogeneous flood population into homogeneous sub-populations (task 5) and also to examine the presence of mixed populations.

4.4.1 Data Base

The USGS records for PDS of 27 streamflow gaging stations in Louisiana were used as the data base in this study. The geographic representation of these selected stations from all across the state is shown in Figure 4.2. However, suitable stations were not found in the coastal area of southern Louisiana and in the alluvial valley of the Mississippi river. The selected flood sequences have at least 30 years of records which are unaffected by major regulations and diversions. The selected period of record of flood sequences were chosen to overlap the same time period of 1950 to 1980 used by the climatologists in defining their hydroclimatic separation of flood events. The pertinent data of those observed flood sequences and watersheds of selected stations are given in Table B.1.

When applying the stochastic flood model to a real world flood process (task 4 and 5), it is important to examine the underlying model assumptions made on

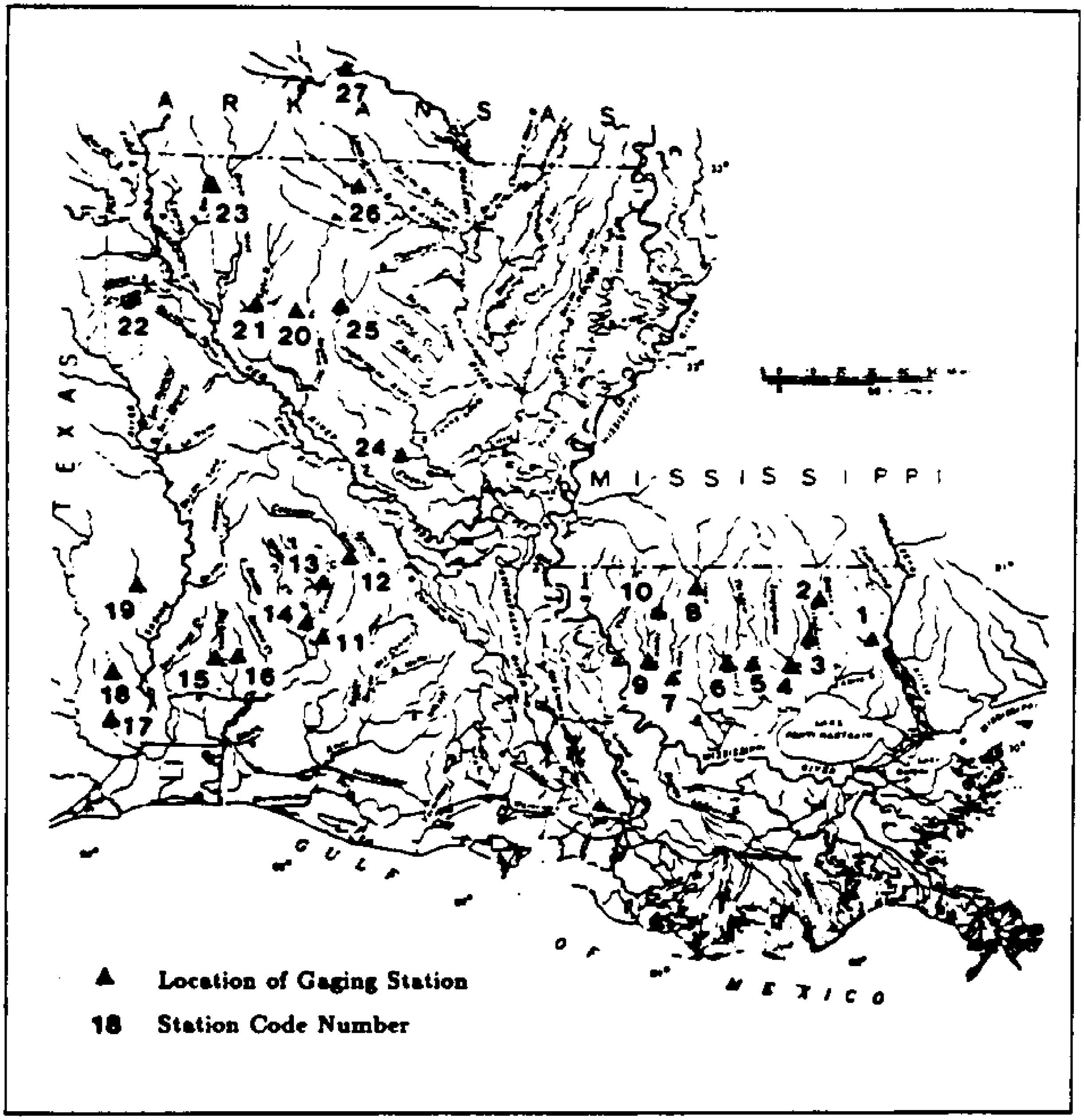


Figure 4.2 Selected Streamflow Gaging Stations in Louisiana.

the observed flood sequences. Therefore, the following steps were employed on the observed flood sequences for each selected gaging station (Cruise and Arora, 1990).

a. The PDS is separated into hydroclimatically homogeneous flood groups with distinct flood generating mechanisms based on climatic information,

b. The Poisson accepted threshold level is found (if possible) for annually homogeneous PDS based on the Poisson statistical test described by Cunnane (1979), and Ashkar and Rousselle (1987),

c. The statistical similarities of hydroclimatically separated flood groups of PDS are tested using the modified Mann-Whitney U statistical test proposed by Halperin (1960) and those statistically similar flood groups are combined into one flood population,

d. The resulting PDS are tested for the exponential hypothesis for magnitude of flood exceedances based on the Kolmogorov (D) and Cramer-Von Mises (w^2) tests described by Stephens (1974), and

e. The same PDS are examined for the feasibility of the Weibull marginal for magnitude of flood exceedances.

4.4.2 Hydroclimatic Separation of Flood Sequences

The hydroclimatic separation procedure groups flood events in a PDS produced by the same type of atmospheric circulation mechanism based on the analysis of daily surface and synoptic weather maps and precipitation isohytal maps. The Figure

4.3 illustrates an example of these maps for flood events under a specific meteorological circulation pattern. A detailed analysis of hydroclimatic separation of flood sequences for several gauging stations in Arizona was described by Hirschboeck (1985, 1987).

The majority of floods on Louisiana streams evolve from a variety of flood generating climatic mechanisms such as widespread frontal, localized convective and disturbed tropical activities (USGS, 1988; and Cruise and Arora, 1990). About 70% of annual rainfall in Louisiana is associated with winter/spring frontal passages. The remaining 30% is produced by summer convective (10%) and summer/fall tropical (20%) related storm events. Frontal activities normally occurring in winter/spring (November through April) and tropical disturbed weather events generally occurring in summer/fall (May through October) are mainly responsible for the majority of flooding in Louisiana. Therefore, in this study it is assumed that floods in Louisiana are seasonally controlled as 'winter/spring' and 'summer/fall' and may be produced by two distinct hydroclimatic sub-populations for each observed flood sequence (step a).

4.4.3 Poisson Test for Threshold Level

The selection of a Poisson accepted threshold level above which streamflows are regarded as floods plays an important role in the underlying model assumptions, such as that $\eta(t)$ describes a Poisson process and ξ_ν ($\nu = 1, 2, \ldots, n$) are independent events. The Poisson statistical test proposed by Cunnane (1979) and described by

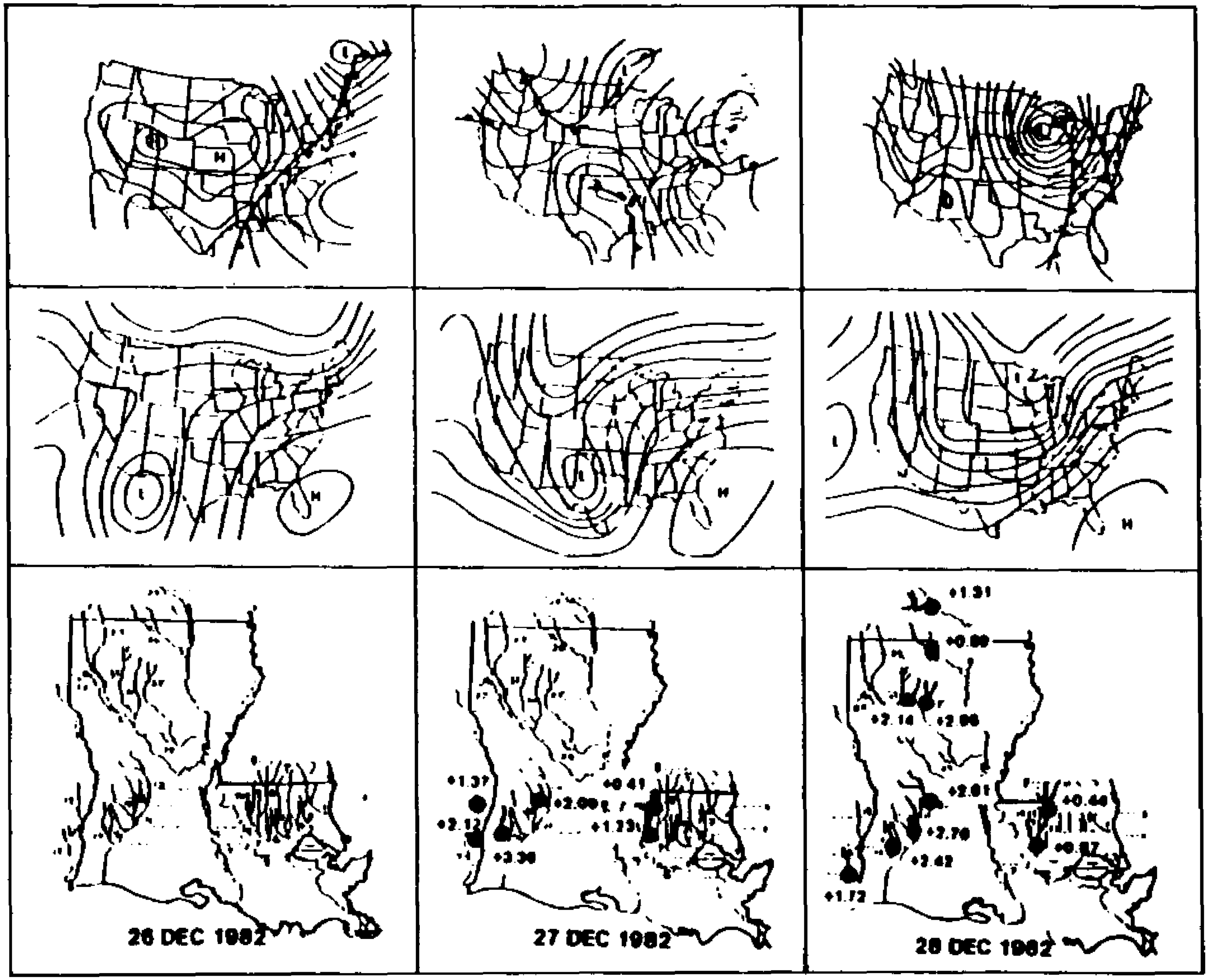


Figure 4.3 Surface and Synoptic Weather Maps and Flooding Sequence in Louisiana (Hirschboeck and Cruise 1990).

✚1.2 Standardized Flood Magnitude ● Stream Gage at Flood Stage

Ashkar and Rousselle (1987) and Cruise and Arora (1990) which is based on the equality of the mean $E[\eta(t)]$ and the variance $Var[\eta(t)]$ of the Poisson distribution was used to select the threshold level of the observed PDS (step *b*). Thus, the fact that the ratio $R = Var[\eta(t)]/E[\eta(t)]$ of the Poisson process approaches unity could be used intuitively to study the behaviour of the random variable $\eta(t)$ for varying threshold levels of the observed PDS. When R is estimated from the observed flood sample for varying threshold levels, the variation of estimates of R can be statistically evaluated on the basis of the Fisher dispersion test statistic $(N_y - 1)R$ which is χ^2 distributed with $(N_y - 1)$ degrees of freedom (Cunnane, 1979). Thus, the critical value of R is obtained as

$$R_c = \chi^2_{\alpha, N_y - 1}/(N_y - 1) \tag{4.6}$$

where

N_y = number of years of records, and

α = significance level of the statistical test (say 10%).

In carrying out this test, starting from the initial USGS threshold level and for small increments above this level, the corresponding ratios of R were estimated and compared to the critical value of R_c given by Eqn. (4.6). The acceptance of the Poisson assumption for $\eta(t)$ of the selected threshold level depends on $R < R_{c,1-\alpha}$ or $R > R_{c,\alpha}$ whether $R > 1$ or $R < 1$, respectively.

The test was carried out on annually homogeneous observed PDS by raising the initial USGS threshold level until either the flood sequence was accepted at the 10% significance level as Poissonian or the mean arrival rate of flood events was reduced to one event per year. When the flood series has less than one event per year, the PDS model loses its advantages over the AFS model in terms of sampling error. The results of the Poisson statistical test on annually homogeneous PDS of all sites are given in Table B.2. Those selected Poisson admissible threshold levels for annually homogeneous PDS can also be used in the subsequent analysis of annually nonhomogeneous PDS utilizing the regenerative property of the Poisson distribution (Cunnane, 1979).

4.4.4 Mann-Whitney Test for Mixed Populations

A detailed separation of a flood sequence into its hydroclimatically distinct flood groups is useful for developing a physically based approach to the stochastic flood model. However, such a detailed separation increases the number of model parameters and of course, may not be necessary for stochastically analyzing model predictions. Therefore, in minimizing the number of model parameters, those hydroclimatically distinct flood groups were tested for statistical similarities (step c). At this stage of the analysis, the distributions of the flood groups are not known, therefore, a non-parametric statistical test is required to test the statistical similarities. Hence, the modified Mann-Whitney U statistical test, described by Halperin (1960) was used to test the statistical similarities of two flood groups truncated at the same threshold level. When both flood groups of x and y are truncated at a common threshold level (Q_0) as of annually homogeneous PDS, the modified Mann-Whitney U statistic is computed as

$$U_c = U(n_1, n_2) + (m_1 - n_1)n_2 \tag{4.7}$$

where

n_1 = uncensored sample size of x ($x \geq Q_0$),

n_2 = uncensored sample size of y ($y \geq Q_0$),

r_1 = censored sample size of x ($x \leq Q_0$),

r_2 = censored sample size of y ($y \leq Q_0$),

$m_1 = n_1 + r_1$ = original sample size of x, and

$m_2 = n_2 + r_2$ = original sample size of y.

$U(n_1, n_2)$, the usual Mann-Whitney U statistic is determined as follows;

$$U(n_1, n_2) = \sum_{i=1}^{i=n_1} \sum_{j=1}^{j=n_2} Z_{ij} \tag{4.8}$$

where

$Z_{ij} = 1 \ldots$ if $x_i < y_j$,

$Z_{ij} = 0 \ldots$ if $x_i > y_j$ and

x_i and y_j are flood events of sample x and y, respectively.

When both uncensored sample sizes are greater than 8, the modified U statistic (U_c) is well approximated by the standard normal distribution as

$$Z_{U_c} = \frac{U_c - \mu_{U_c}}{\sigma_{U_c}} \tag{4.9}$$

where

$$\begin{aligned}
\mu_{U_c} &= \frac{m_1 m_2 (m_1 + m_2 - r)(m_1 + m_2 + r - 1)}{2(m_1 + m_2)(m_1 + m_2 - 1)} \\
{\sigma_{U_c}}^2 &= \frac{m_1 m_2 (m_1 + m_2 - r)}{4(m_1 + m_2)}(A + B + C + D - E)
\end{aligned}$$

and

$$
\begin{aligned}
r &= r_1 + r_2 \\
A &= \frac{(m_1 + m_2 - r)^2 - 1}{3(m_1 + m_2 - 1)} \\
B &= \frac{(m_1 - 1)(m_1 + m_2 - r - 1)}{(m_1 + m_2 - 1)} \left[m_2 + \frac{r(2m_2 + 1)}{(m_1 + m_2 - 2)} \right] \\
C &= \frac{(m_1 - 1)(m_1 + m_2 - r - 1)}{(m_1 + m_2 - 1)} \left[\frac{r(r - 1)(m_2 - 1)}{(m_1 + m_2 - 2)(m_1 + m_2 - 3)} \right] \\
D &= m_2 + \frac{r(2m_2 + 1)}{(m_1 + m_2 - 1)} + \frac{r(r - 1)(m_2 - 1)}{(m_1 + m_2 - 1)(m_1 + m_2 - 2)} \\
E &= \frac{m_1 m_2 (m_1 + m_2 - r)(m_1 + m_2 + r - 1)^2}{(m_1 + m_2)(m_1 + m_2 - 1)^2}
\end{aligned}
$$

According to Bradley (1968), this non-parametric test on the test statistic Z_{U_c} has been shown to be powerful even against an alternative parametric test for two known normal distributions with different means. Although the consistency of the standard normal assumption on Z_{U_c} was found to be reduced for small sample sizes; the assumption is considered to be valid even at significance levels as low as 2.5%, when large differences between sample sizes exist, i.e., $n_1 = 5n_2$ (Bradley, 1968).

The acceptance or rejection of the hypothesis of statistical similarity of two flood groups is determined at a given significance level α (say at 10%) depending on whether $Z_{U_c} \leq Z_{1-\alpha/2}$ or $Z_{U_c} \geq Z_{1-\alpha/2}$, respectively. The flood groups, which have been found as statistically similar were combined into one flood population. The results of the modified Mann-Whitney U statistical test at each site are shown in Table B.3.

4.4.5 Exponential Test for Marginal Distribution

The relative inflexibility of the one parameter exponential marginal distribution in fitting to observed flood data makes the quantile estimates very sensitive to its selected threshold level (Cruise and Arora, 1990). Therefore, the exponential hypothesis for the marginal distribution was tested (step d) using two powerful empirical distribution function (EDF) tests such as the Kolmogorov (D) and the Cramer-Von Mises (w^2) described by Stephens (1974).

In testing the exponential hypothesis for a given random sample arranged in an ascending order as $x_1 \le x_2 \le \ldots x_n$, the EDF test statistics (D and w^2) are computed as follows (Stephens, 1974):

Kolmogorov Test Statistic D:

This test statistic D is defined on the basis of the maximum deviation of probabilities between empirical estimates of observed variates and exponentially distributed observed variates.

$$D = max(D^+, D^-) \tag{4.10}$$

where

$$\begin{aligned} D^+ &= max[(i/n) - F(x_i)] \\ D^- &= max[F(x_i) - (i-1)/n] \end{aligned}$$

Cramer-Von Mises Test Statistic w^2:

The test statistic w^2 is defined on the basis of the sum of the maximum deviation of probabilities between empirical estimates of observed variates and exponentially distributed observed variates.

$$w^2 = \sum_{i=1}^{n}[F(x_i) - (2i-1)/2n]^2 + (1/12n) \tag{4.11}$$

where

$$\begin{aligned} F(x_i) &= 1 - exp(\hat{\beta}x_i) \\ \hat{\beta} &= 1/(1/n)\sum_{i=1}^{n} x_i \end{aligned}$$

The asymptotic critical values for both test statistics at different significance levels given by Stephens (1974) were used to test the exponential hypothesis. This hypothesis was tested at 5% significance level by raising the initial USGS threshold level (if necessary) until the mean arrival rate of flood events was reduced to one event per year. The results of these EDF tests for all observed flood sequences are given in Table B.4.

4.4.6 Fitting of Exponential and Weibull Models

In analyzing the descriptive properties (tasks 4 and 5), both the proposed (Weibull as marginal) and traditional (exponential as marginal) exact extreme value stochastic flood models were fitted to the observed flood sequences. Both simple and compound formulations of those models were examined for purpose of comparison, even if all flood events in an observed flood sequence were shown to belong to a statistically similar single flood population. The descriptive properties of both simple and compound formulations of the exponential and Weibull models were measured by the standardized root mean square error (SRMSE) between each model prediction and the observed annual flood sequence and computed as follows;

$$SRMSE = \left\{ \frac{1}{n} \sum_{i=1}^{n} \left(\frac{\hat{x}_i - x_i}{\bar{x}} \right)^2 \right\}^{1/2} \tag{4.12}$$

where

$\bar{x}$ = mean of the observed flood events x_i,

x_i = observed value of i^{th} flood event in the sequence, and

$\hat{x}_i$ = model predicted value of i^{th} flood event with same probability as x_i and was computed as follows:

$$\hat{x}_i = F^{-1}[p(x_i)] = F^{-1}\left(\frac{m_i}{N_y + 1}\right) \tag{4.13}$$

where

$p(x_i)$ = Weibull plotting position of x_i,

m_i = rank of x_i in descending order, and

N_y = number of years of records of the flood sequence.

The results of the fittings of the exponential and Weibull models are given in Table B.5 for annually homogeneous data and Table B.6 for annually nonhomogeneous data.

Chapter 5

Results and Discussion

5.1 Introduction

The purpose of this chapter is to analyze the results obtained when performing the major tasks 1 through 5 described in the previous chapter. In analyzing the sampling experimental results (tasks 1 to 3), the asymptotic properties of performance indices including $BIAS$ and $RMSE$ of quantile estimation were examined. Three representative sample sizes of 50, 100 and 1000 were selected for demonstration purposes as often they are of interest to engineers in analyzing hydrologic frequency models based on AFS, PDS and regional data, respectively. Therefore, only for sample sizes 50, 100 and 1000, the asymptotic properties of $BIAS$ and $RMSE$ of quantile estimation are shown here in performing task 1 to 3 but complete results are shown in Appendix A.

In the case of tasks 4 and 5, the results obtained were discussed on the basis of SRMSE between both the simple and compound formulations of the exponential and Weibull model predictions and the annual flood series for each selected gaging station in Louisiana.

5.2 Analysis of Predictive Properties

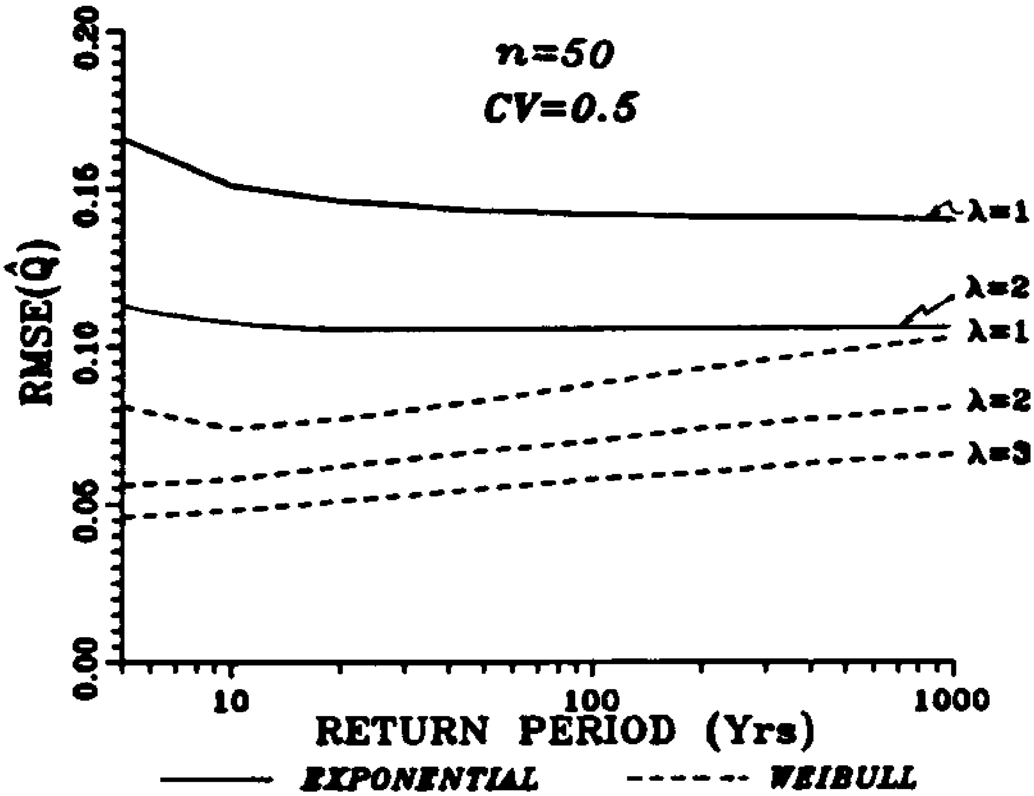
5.2.1 Task 1: Exponential Vs. Weibull Model

The results of $BIAS$ and $RMSE$ analysis for exponential and Weibull quantile estimations are given in Tables A.1 to A.6 and Tables A.7 to A.30, respectively. These results were used to examine the comparative performance of the simple Weibull model to that of the exponential model for a wide range of population CV (0.5 to 3.0) and sample sizes of 50, 100 and 1000.

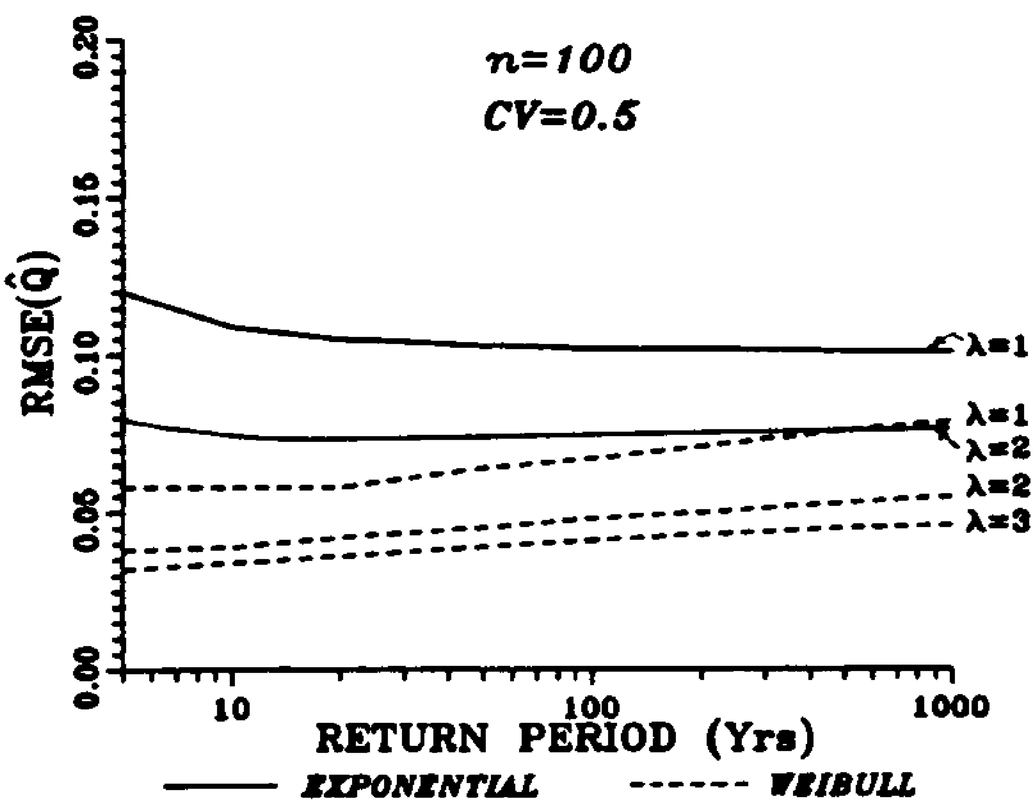
As shown in Figure 5.1, when $CV = 0.5$ the Weibull model consistently shows better performance relative to the exponential model in terms of $RMSE$ of all quantile estimation over the entire range of sample sizes tested. In the case of $CV = 1.0$ as seen from Figure 5.2, at Poisson $\lambda = 2$, the Weibull model still performs well in terms of $RMSE$ of quantile estimation of up to about 100 yr return period, even for sample size of 50. As the sample size increases ($n \geq 100$), the performance of the Weibull model shows further improvements over the exponential model for over 100 yr return periods. However, as shown in Figure 5.3, when $CV = 1.5$ the performance of the Weibull model begins to deteriorate. In this case only at Poisson $\lambda = 3$, the performance of the Weibull model is comparable to that of the exponential model but only for fairly small return periods. Figure 5.4 clearly shows the complete deterioration of prediction power of the Weibull model relative to the exponential model for $CV = 3.0$.

These results suggest that for samples having $CV < 1.5$, the Weibull model is a better alternative to the exponential model for samples containing as few as two flood events ($\lambda = 2$) per year. Interestingly enough, as seen from Tables B.2 and B.3, both of these population conditions are shown to exist in most of the observed flood series of Louisiana. However, when $CV \geq 1.5$ the Weibull model may not be a better alternative even for samples having as many as three floods events ($\lambda = 3$) per year.

As seen from Table 5.1, a summary of $BIAS$ analysis for exponential and Weibull model quantile estimations, when sample size $n \leq 50$ quantile estimates of the Weibull model generally shows larger positive $BIAS$ for $1.5 \leq CV \leq 3.0$ and smaller negative $BIAS$ for $0.5 \leq CV < 1.5$ compared to that of the exponential model. However, as the sample size increases $BIAS$ of quantile estimates of the Weibull model is comparable to the exponential model for all CV values tested. This shows that particularly for small samples, the quantile estimates of the Weibull model are strongly dependent on the parameter estimation method unlike the one parameter exponential counterpart.


n=50
CV=0.5
RMSE($\hat{Q}$)
0.00
0.05
0.10
0.15
0.20
10
100
1000
RETURN PERIOD (Yrs)
λ=1
λ=2
λ=1
λ=2
λ=3
EXPONENTIAL
WEIBULL

(a)


n=100
CV=0.5
RMSE($\hat{Q}$)
0.00
0.05
0.10
0.15
0.20
10
100
1000
RETURN PERIOD (Yrs)
λ=1
λ=1
λ=2
λ=2
λ=3
EXPONENTIAL
WEIBULL

(b)

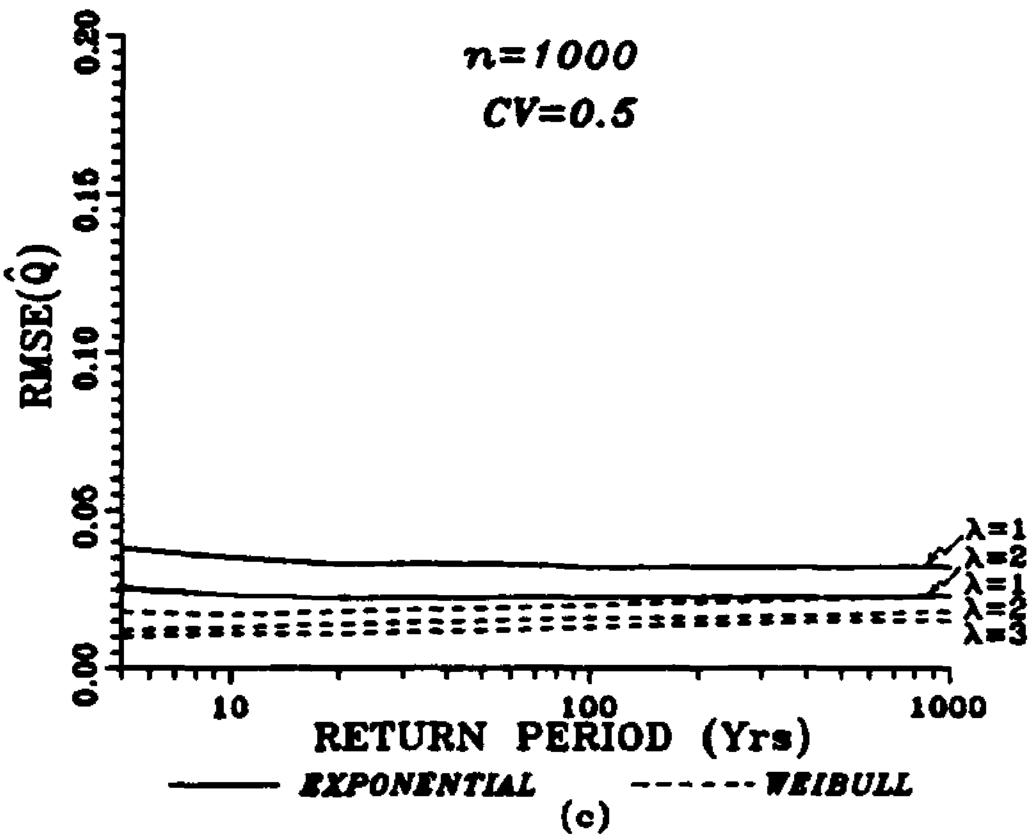
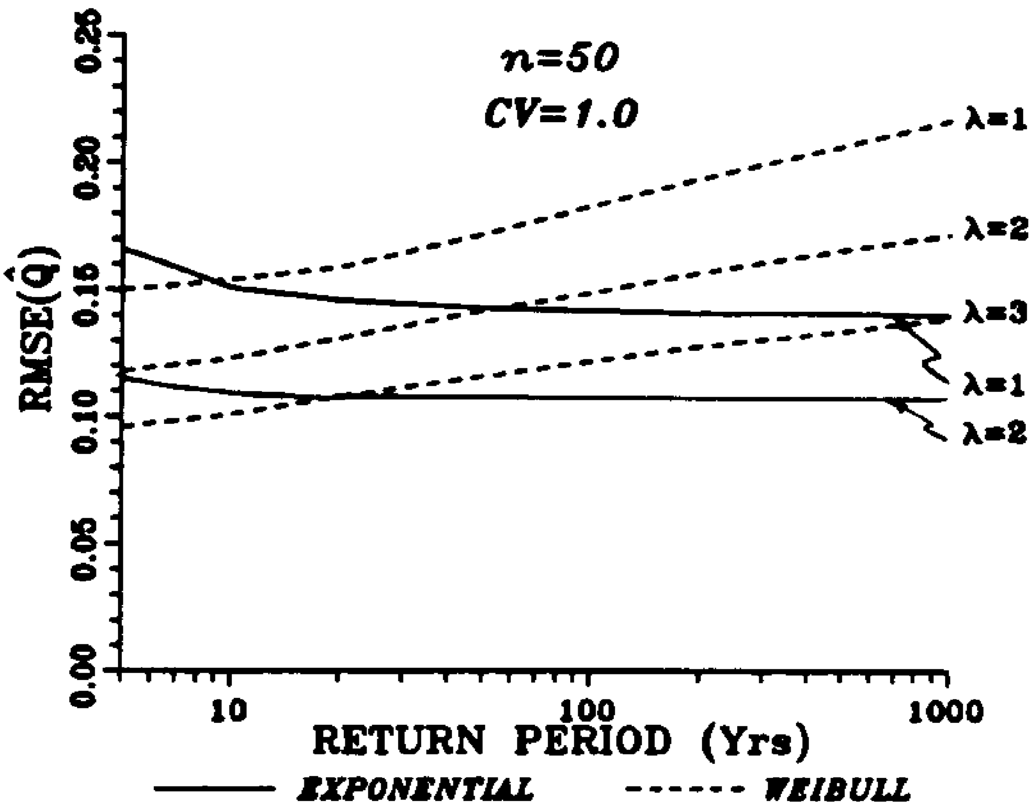
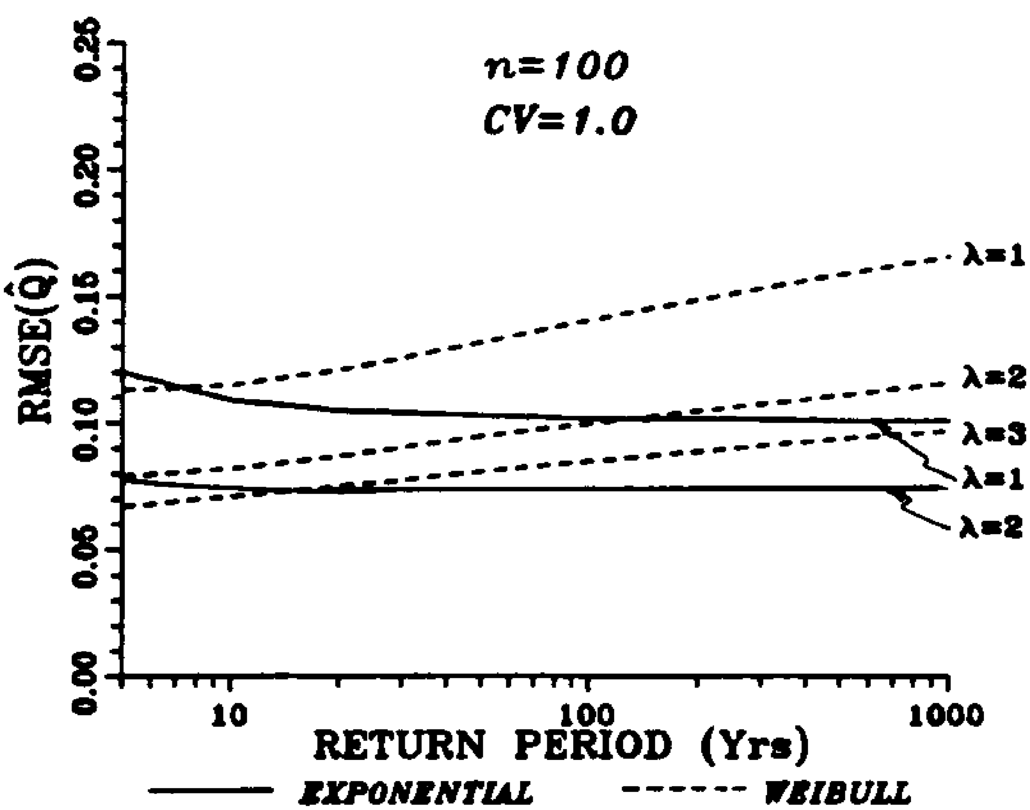


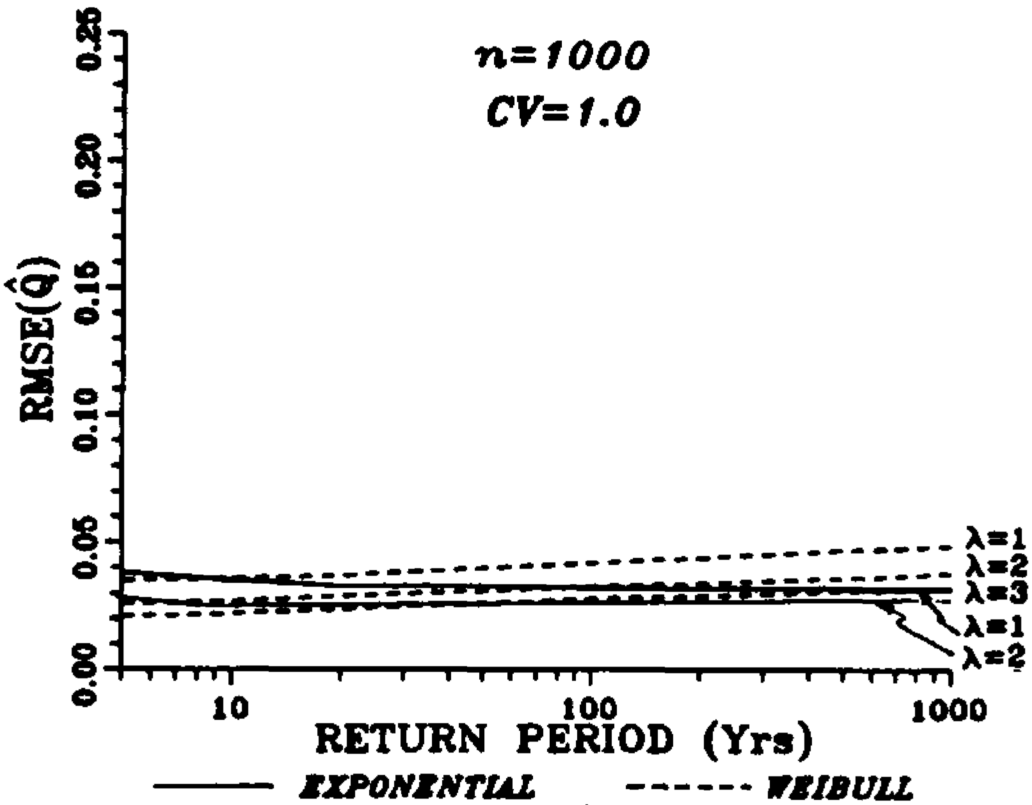
Figure 5.1 Performance of the Weibull Model Relative to the Exponential Model: For Varying $\lambda=1, 2, 3$ at $CV=0.5$.
(a). Sample Size: 50, (b) Sample Size: 100 and (c). Sample Size: 1000


n=50
CV=1.0
λ=1
λ=2
λ=3
λ=1
λ=2
RMSE(Q̂)
0.00
0.05
0.10
0.15
0.20
0.25
10
100
1000
RETURN PERIOD (Yrs)
EXPONENTIAL
WEIBULL

(a)


n=100
CV=1.0
λ=1
λ=2
λ=3
λ=1
λ=2
RMSE(Q̂)
0.00
0.05
0.10
0.15
0.20
0.25
10
100
1000
RETURN PERIOD (Yrs)
EXPONENTIAL
WEIBULL

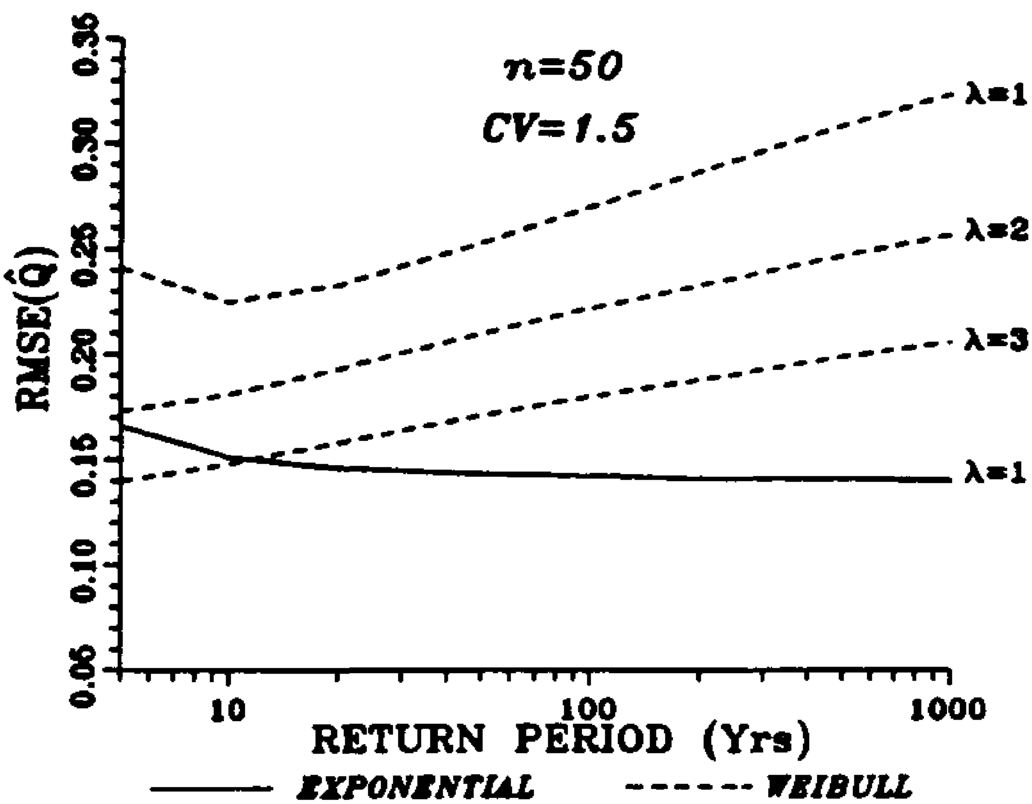
(b)



(c)

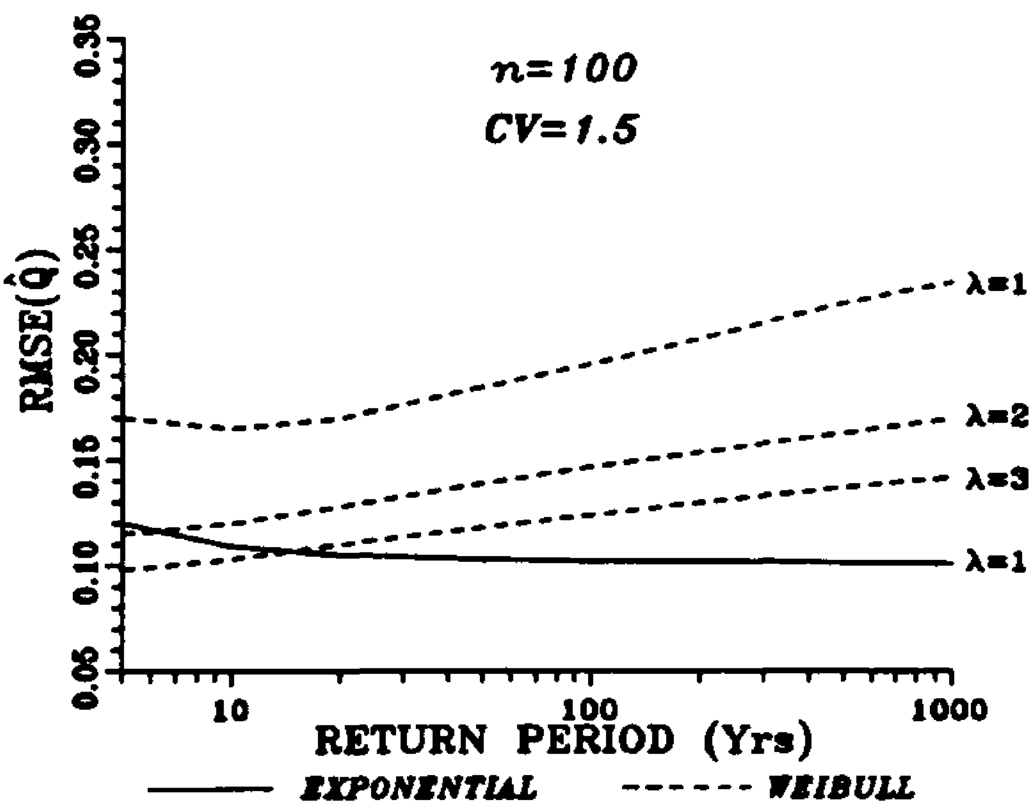
Figure 5.2 Performance of the Weibull Model Relative to the Exponential Model: For Varying λ=1, 2, 3 at CV=1.0.

(a). Sample Size: 50, (b) Sample Size: 100 and (c). Sample Size: 1000



n=50
CV=1.5
λ=1
λ=2
λ=3
λ=1
RMSE(Q̂)
0.05
0.10
0.15
0.20
0.25
0.30
0.35
10
100
1000
RETURN PERIOD (Yrs)
EXPONENTIAL
WEIBULL

(a)



n=100
CV=1.5
λ=1
λ=2
λ=3
λ=1
RMSE(Q̂)
0.05
0.10
0.15
0.20
0.25
0.30
0.35
10
100
1000
RETURN PERIOD (Yrs)
EXPONENTIAL
WEIBULL

(b)

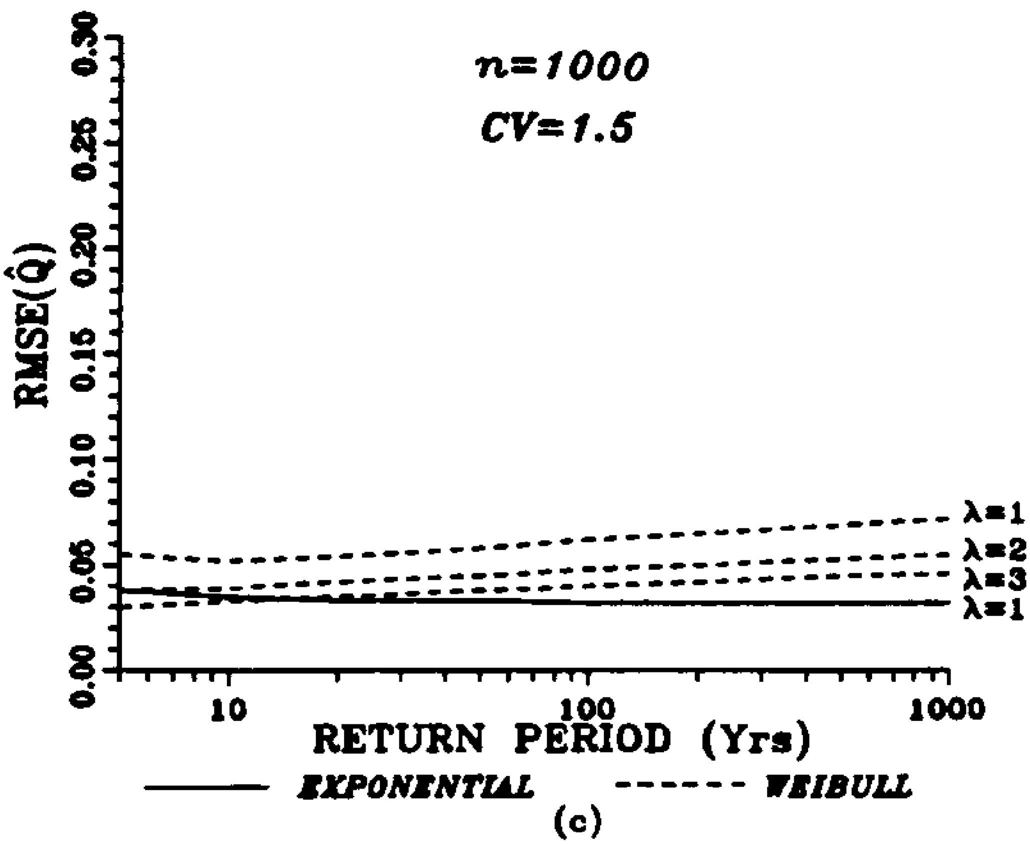
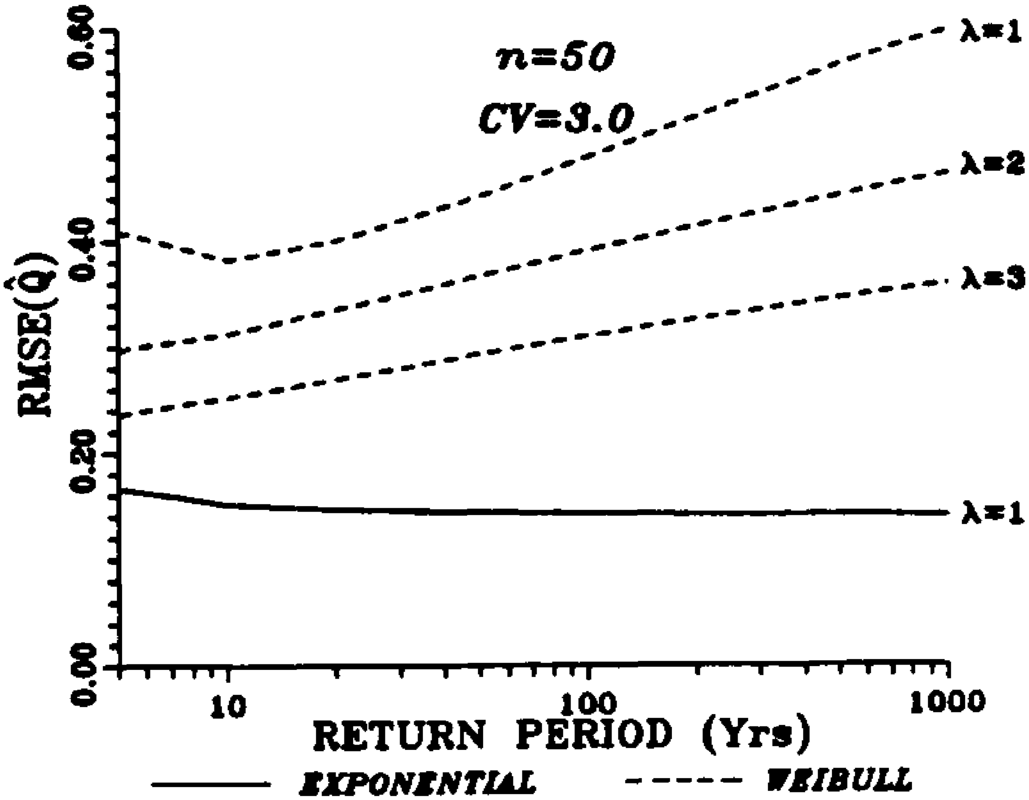
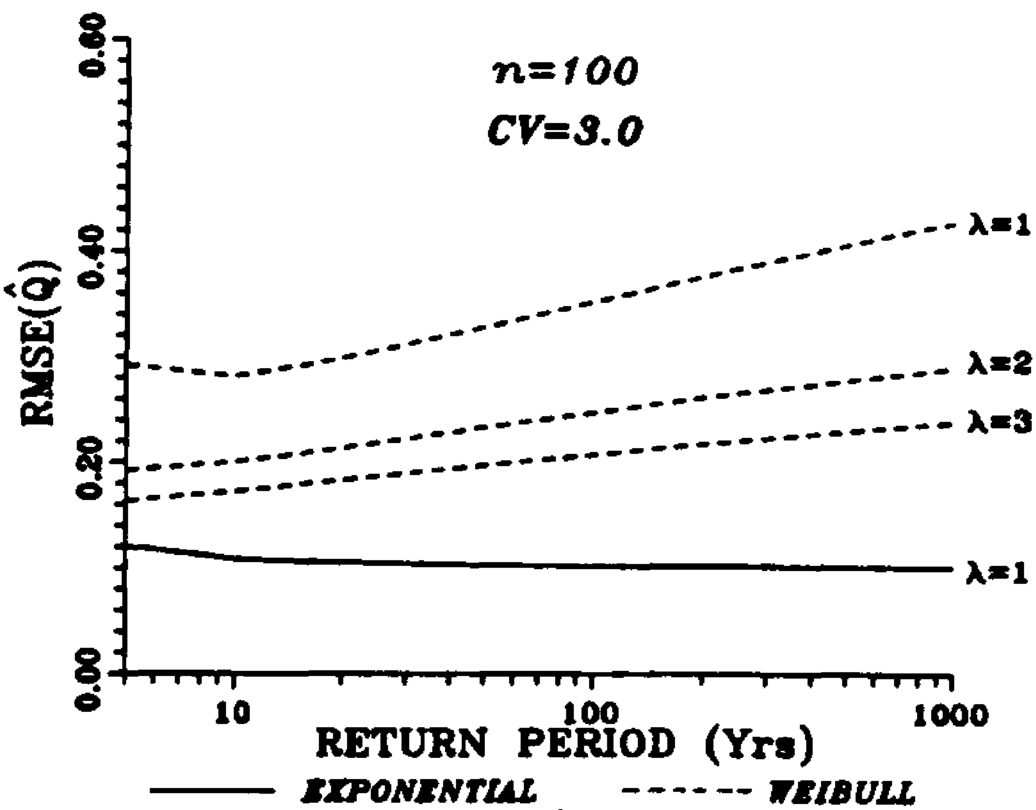


Figure 5.3 Performance of the Weibull Model Relative to the Exponential Model: For Varying λ=1, 2, 3 at CV=1.5.

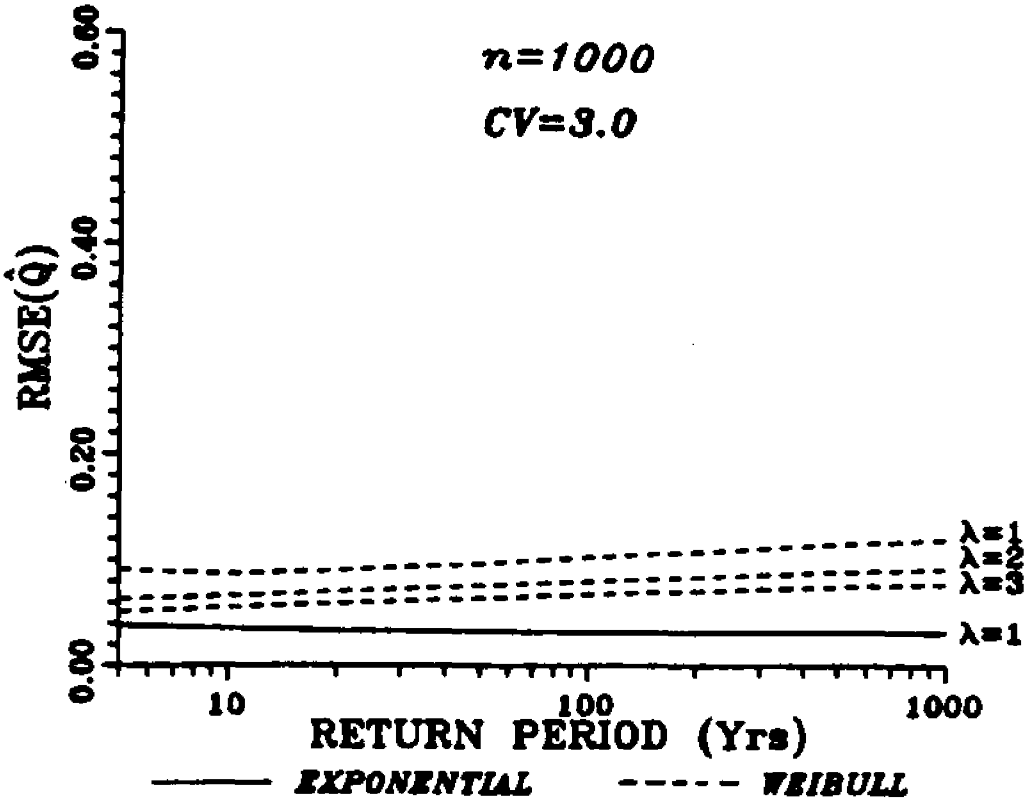
(a). Sample Size: 50, (b) Sample Size: 100 and (c). Sample Size: 1000


n=50
CV=3.0
RMSE(Q̂)
0.00
0.20
0.40
0.60
10
100
1000
RETURN PERIOD (Yrs)
λ=1
λ=2
λ=3
λ=1
EXPONENTIAL
WEIBULL

(a)


n=100
CV=3.0
RMSE(Q̂)
0.00
0.20
0.40
0.60
10
100
1000
RETURN PERIOD (Yrs)
λ=1
λ=2
λ=3
λ=1
EXPONENTIAL
WEIBULL

(b)



(c)

Figure 5.4 Performance of the Weibull Model Relative to the Exponential Model: For Varying λ=1, 2, 3 at CV=3.0.
(a). Sample Size: 50, (b) Sample Size: 100 and (c). Sample Size: 1000

Table 5.1 Summary of $BIAS$ of Selected Quantile Estimates

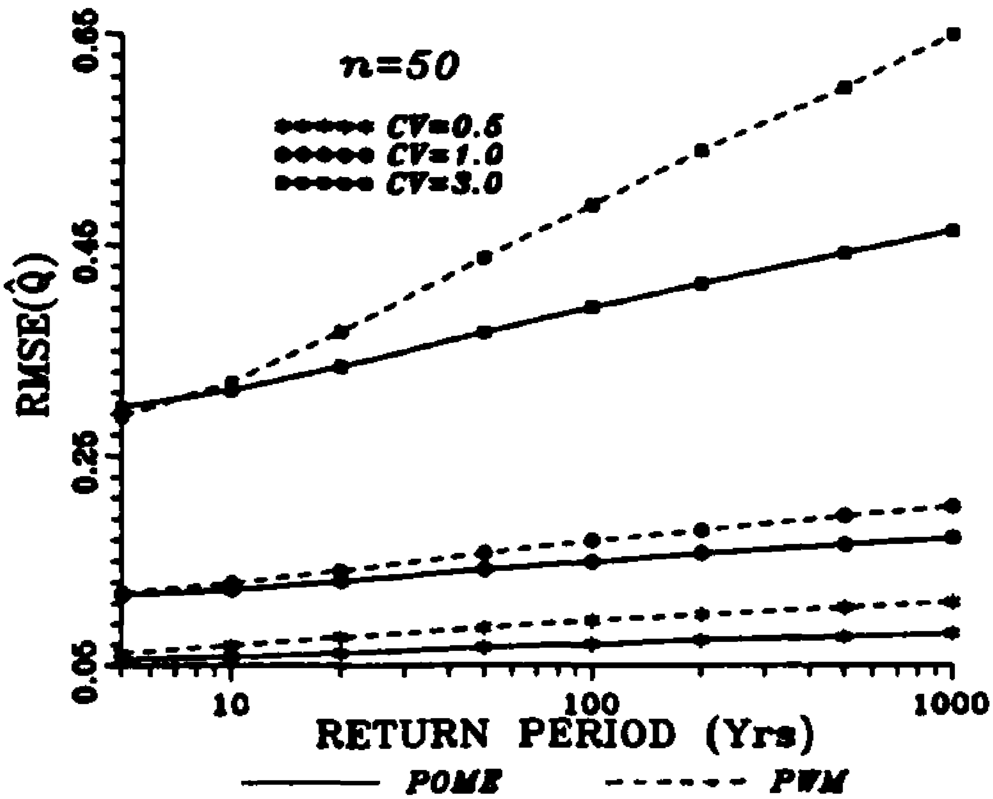
Sample Size n	Return Period T	Expon. Model $\lambda = 1$	Weibull Model $CV = 0.5$ $\lambda = 1$	$CV = 0.5$ $\lambda = 2$	$CV = 1.5$ $\lambda = 1$	$CV = 1.5$ $\lambda = 2$	$CV = 3.0$ $\lambda = 1$	$CV = 3.0$ $\lambda = 2$
	10	-0.009	-0.011	0.000	-0.016	0.009	0.001	0.034
50	100	-0.007	-0.013	0.000	-0.015	0.017	0.015	0.054
	1000	-0.006	-0.013	0.001	-0.008	0.023	0.044	0.075
	10	-0.004	-0.005	-0.004	-0.006	-0.006	0.005	0.003
100	100	-0.002	-0.006	-0.004	-0.005	-0.004	0.015	0.005
	1000	-0.001	-0.007	-0.004	-0.004	-0.002	0.031	0.012
	10	-0.001	-0.001	0.000	-0.002	0.000	-0.001	0.001
1000	100	-0.001	-0.001	0.000	-0.001	0.000	0.000	0.002
	1000	-0.001	-0.001	0.000	0.000	0.001	0.002	0.003

5.2.2 Task 2: POME Vs. PWM Estimator

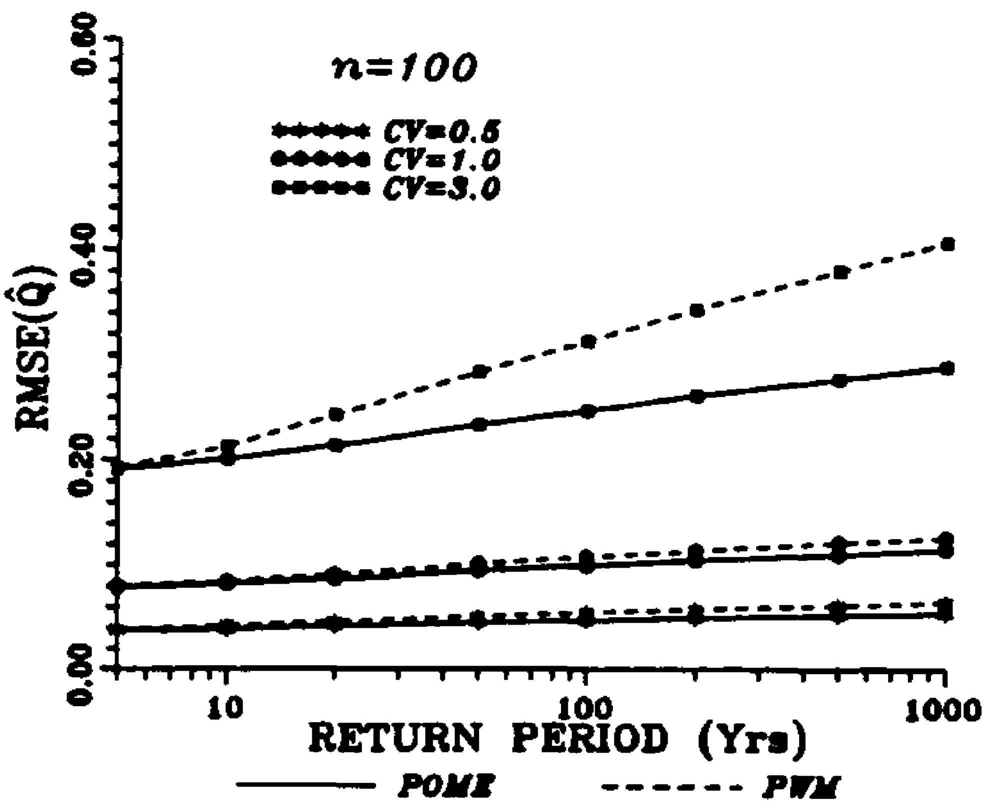
The results of *BIAS* and *RMSE* analysis for Weibull quantile estimation by the POME and PWM are given in Tables A.7 to A.30 and Tables A.31 to A.54, respectively. As before, these results were used to analyze the performances of both the POME and PWM estimators for a wide range of *CV* (0.5 to 3.0) and sample sizes of 50, 100 and 1000.

As shown in Figure 5.5, for small *CV*'s (≤ 1.0) the POME estimator consistently performs well compared to the PWM in terms of *RMSE* of all quantile estimation over the entire range of sample sizes tested, except for very large samples ($n \geq 1000$), where it appears that both the POME and PWM estimators show comparable performances. In the case of large *CV*'s (≥ 3.0) and small sample sizes ($n \leq 50$), the PWM estimator demonstrates improving performance over the POME in terms of *RMSE* of quantile estimation for small return periods. However, this improving performance of the PWM over the POME gets reversed as the sample size increases.

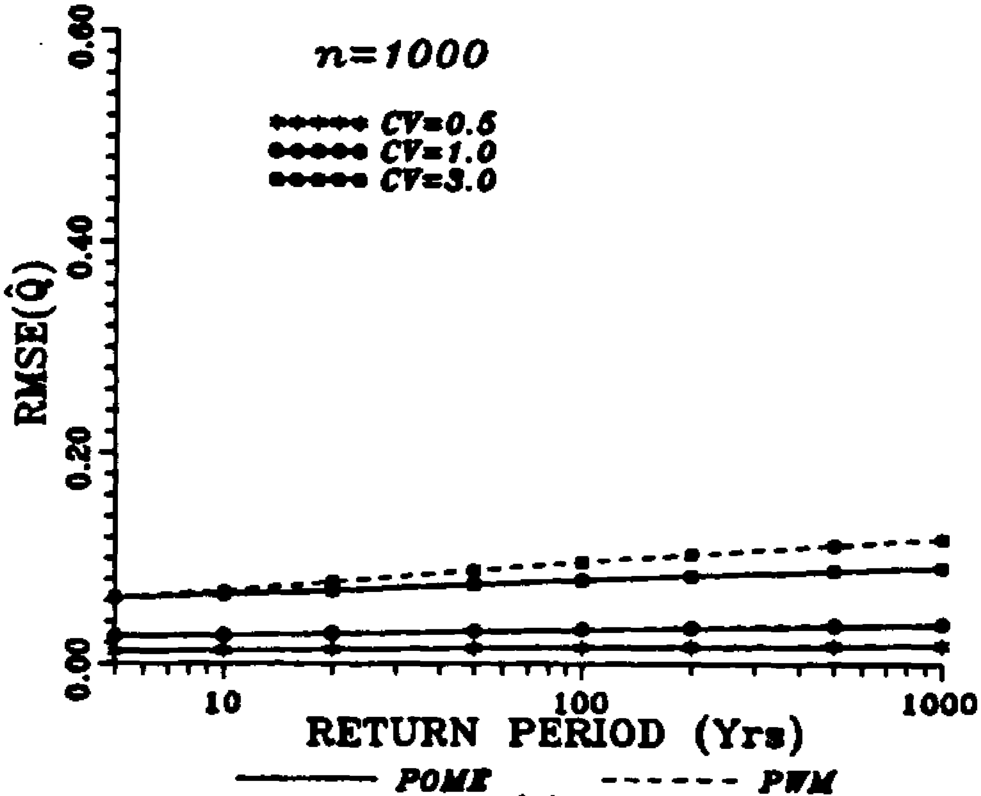
This may have resulted from the decreasing *BIAS* of smaller quantile estimation by the PWM for small samples ($n \leq 50$) with large *CV*'s (≥ 3.0) as seen from Figure 5.6. In the case of small *CV*'s (≤ 1.0), the PWM estimator generally shows larger *BIAS* of quantile estimation for all return periods. The above results of the performance analysis of the POME and PWM in terms of *BIAS* and *RMSE* of quantile estimation, precisely agreed with those reported by Singh et al. (1990).


n=50
CV=0.5
CV=1.0
CV=3.0
0.65
0.45
0.25
0.05
RMSE(Q̂)
10
100
1000
RETURN PERIOD (Yrs)
POME
PWM

(a)


n=100
CV=0.5
CV=1.0
CV=3.0
0.60
0.40
0.20
0.00
RMSE(Q̂)
10
100
1000
RETURN PERIOD (Yrs)
POME
PWM

(b)

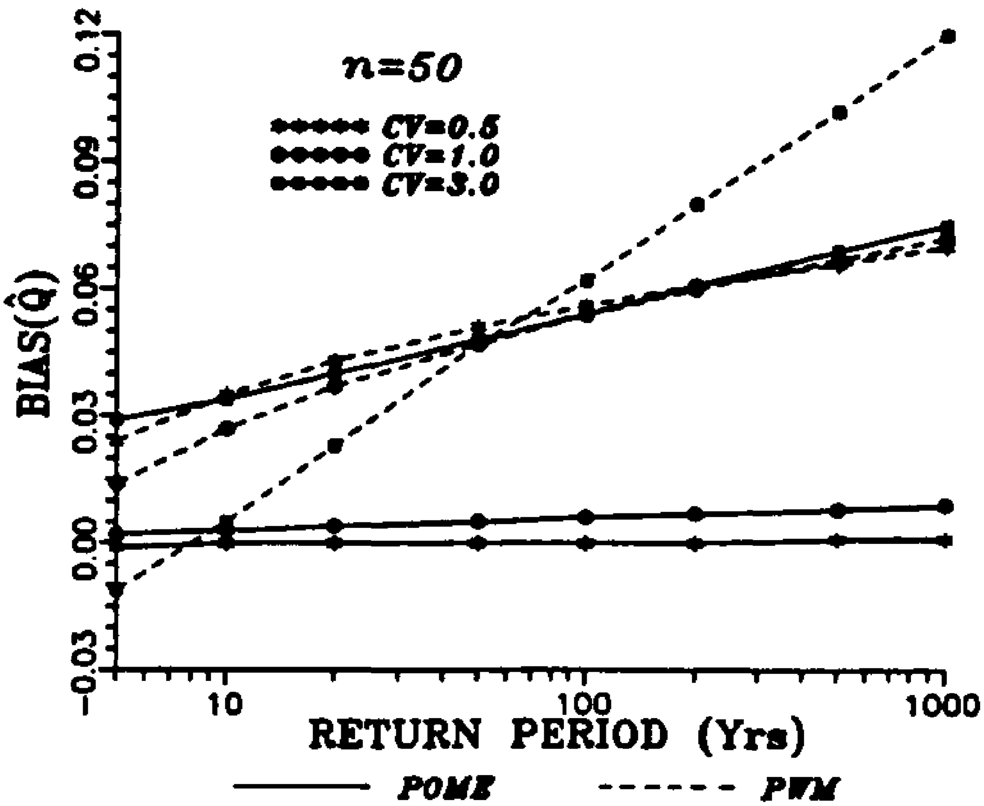


(c)

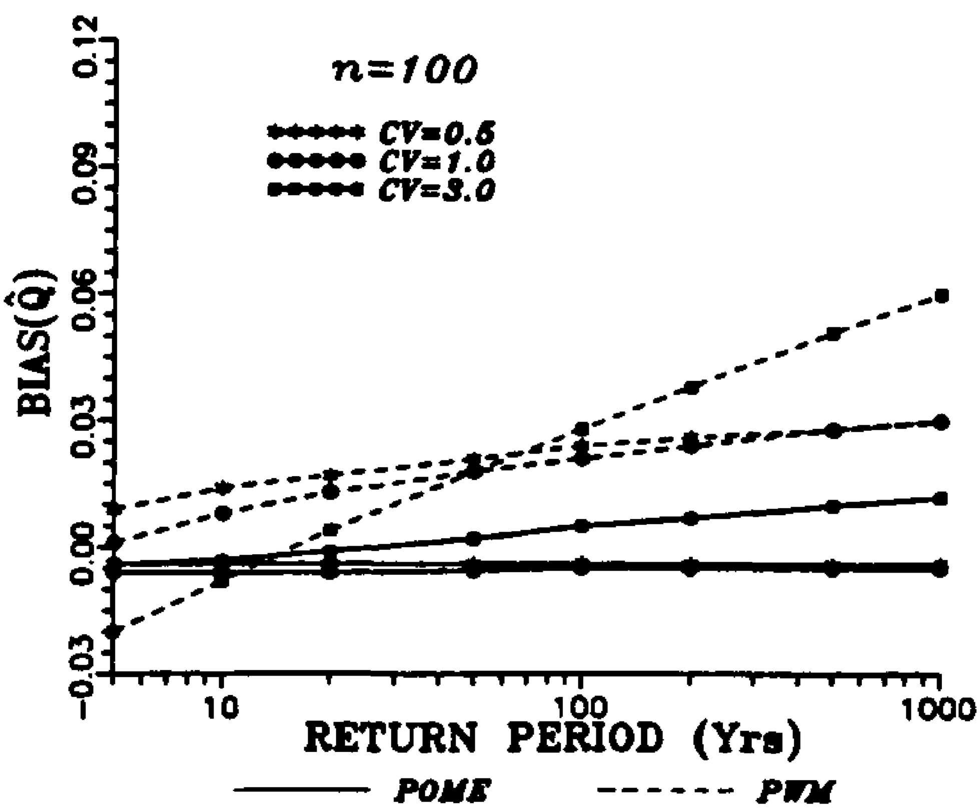
Figure 5.5 Performance of Parameter Estimators of the Weibull Model:

RMSE of Quantiles for POME and PWM.

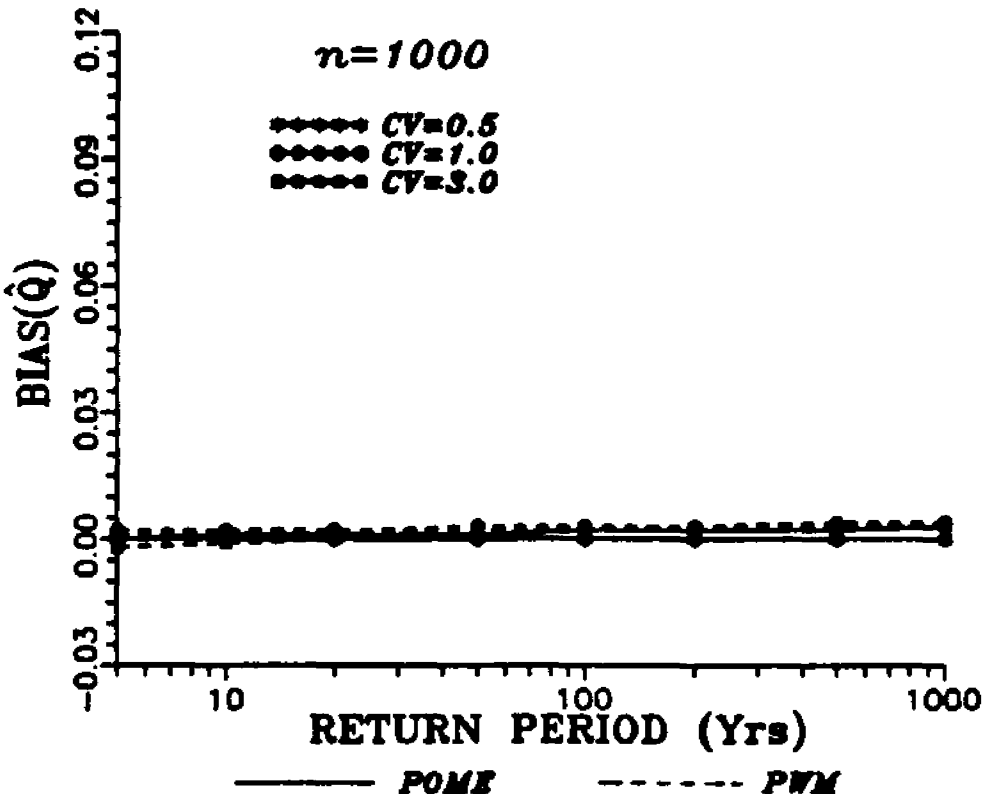
(a). Sample Size: 50, (b) Sample Size: 100 and (c). Sample Size: 1000


n=50
CV=0.5
CV=1.0
CV=3.0
BIAS(Q̂)
0.12
0.09
0.06
0.03
0.00
-0.03
10
100
1000
RETURN PERIOD (Yrs)
POME
PWM

(a)


n=100
CV=0.5
CV=1.0
CV=3.0
BIAS(Q̂)
0.12
0.09
0.06
0.03
0.00
-0.03
10
100
1000
RETURN PERIOD (Yrs)
POME
PWM

(b)



(c)

Figure 5.6 Performance of Parameter Estimators of the Weibull Model:
BIAS of Quantiles for POME and PWM.
(a). Sample Size: 50, (b) Sample Size: 100 and (c). Sample Size: 1000

5.2.3 Task 3: Separated Vs. Mixed Sub-Populations

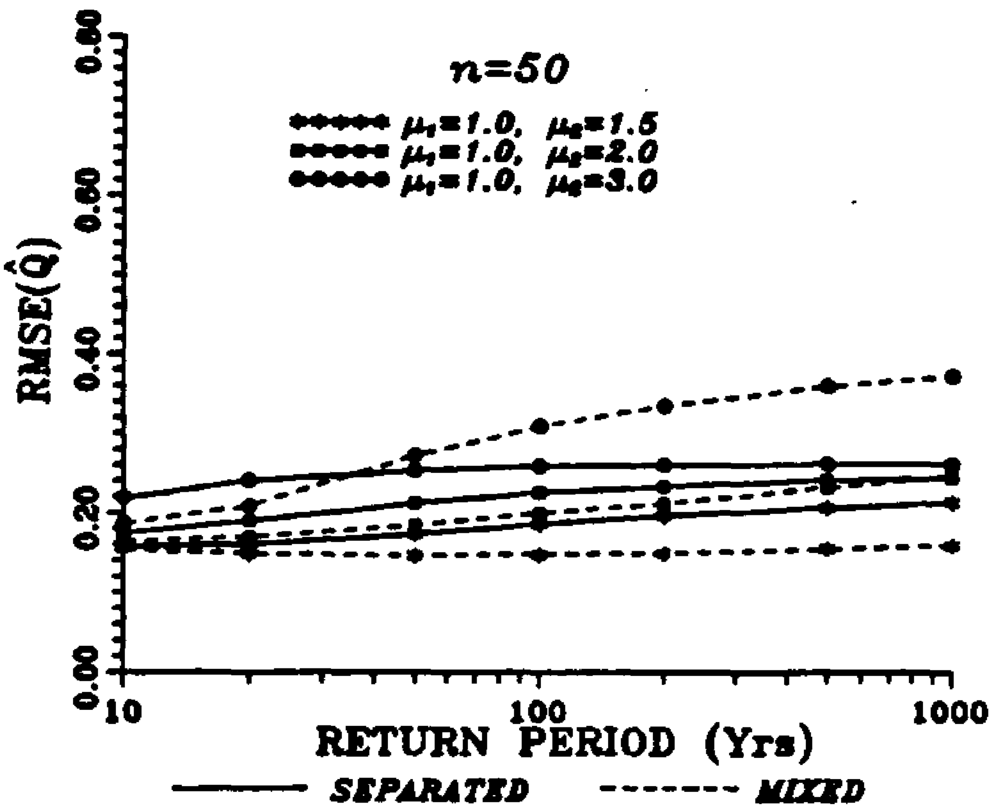
For Exponential Model:

The results of $BIAS$ and $RMSE$ for quantile estimation analysis of nonhomogeneous population, based on the compound exponential model and the simple exponential model are given in Tables A.55 to A.72 and Tables A.73 to A.90, respectively. These results were used to examine the comparative performance of the compound exponential model relative to the simple exponential model. In analyzing the effect of degree of nonhomogeneity between the sub-populations on the performances of the compound and simple models in terms of means of the sub-populations, the ratio of means (μ_2/μ_1) of the two sub-populations was allowed to vary from 1.5 to 3.0.

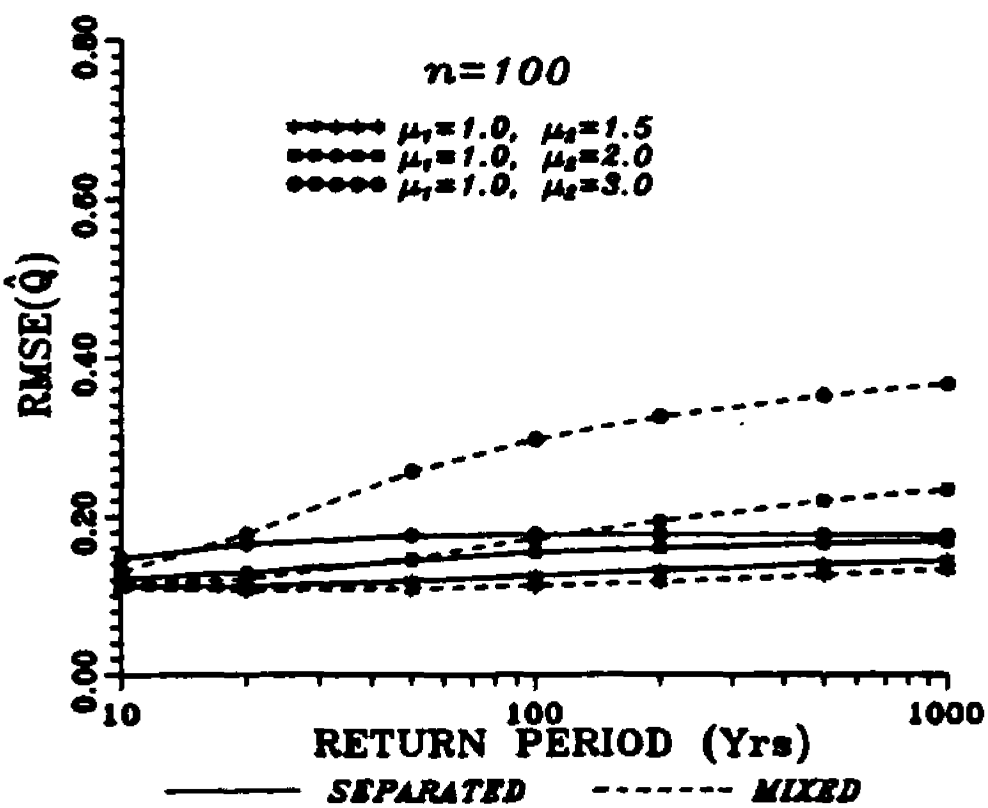
As seen from Figure 5.7, for the small ratio of means $\mu_2/\mu_1 = 1.5$ and sample sizes of 50 and 100, the simple model (sub-populations are mixed) demonstrates better performance relative to the compound model (sub-populations are separated) in terms of $RMSE$ of all quantile estimation. However, as the sample size increases the performance of the simple model slowly deteriorates, such that when the sample size is very large ($n = 1000$) the compound model takes the place of the simple model. For the average ratio of means $\mu_2/\mu_1 = 2.0$ and sample size of 50, the simple model shows only a marginally better performance over the compound model. However, with increasing sample size the compound model rapidly gains its superiority over the simple model and performs well for return periods of over 100 yrs. In the case of large ratio of means ($\mu_2/\mu_1 = 3.0$), even for sample of size 50, the compound model shows superior performance over the simple model for return periods as low as 25 years. As the sample size increases, the superior performance of the compound model shows further improvements for all return periods.

As shown in Figure 5.8, for all sample sizes tested the simple model where the sub-populations are mixed always shows larger negative $BIAS$ of all quantile estimation relative to the compound model. It also demonstrates that this negative $BIAS$ rapidly increases as the ratio of means (μ_2/μ_1) of the sub-populations increases. Conversely, the compound model in which the sub-populations are separated shows very small or zero $BIAS$ of all quantile estimation, irrespective of the ratio of means of the sub-populations and sample sizes tested.

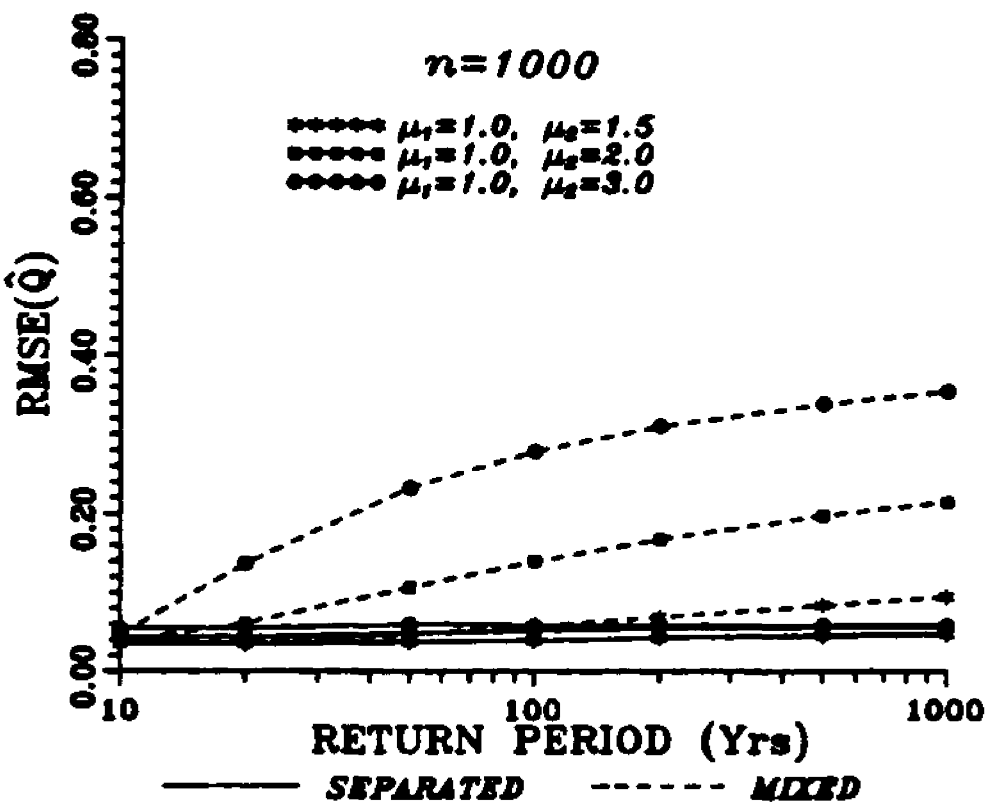
This exercise demonstrates that the simple model always under-estimates the quantile values for mixed populations, where two or more sub-populations with different means are mixed. The under-estimation of flood quantiles increases further as the degree of nonhomogeneity (in terms of means) of sub-populations increases. Therefore, to describe mixed populations the compound model formulation, in which the sub-populations are separated, would be recommended.


n=50
μ1=1.0, μ2=1.5
μ1=1.0, μ2=2.0
μ1=1.0, μ2=3.0
RMSE(Q̂)
0.00
0.20
0.40
0.60
0.80
10
100
1000
RETURN PERIOD (Yrs)
SEPARATED
MIXED

(a)

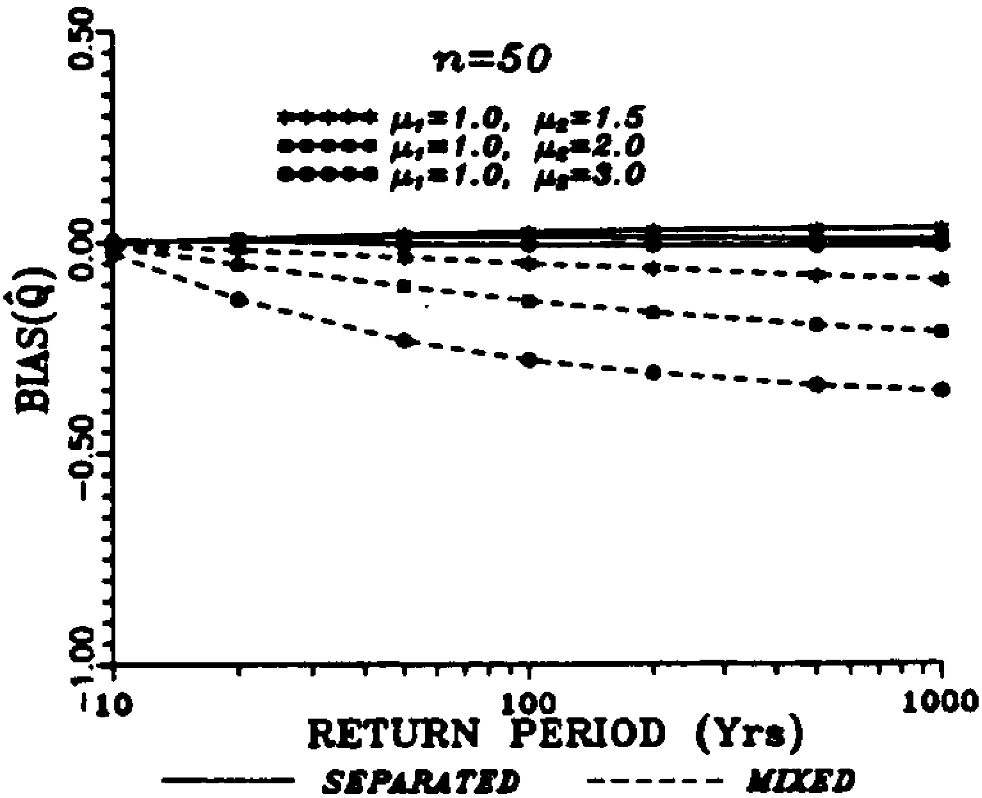

n=100
μ1=1.0, μ2=1.5
μ1=1.0, μ2=2.0
μ1=1.0, μ2=3.0
RMSE(Q̂)
0.00
0.20
0.40
0.60
0.80
10
100
1000
RETURN PERIOD (Yrs)
SEPARATED
MIXED

(b)



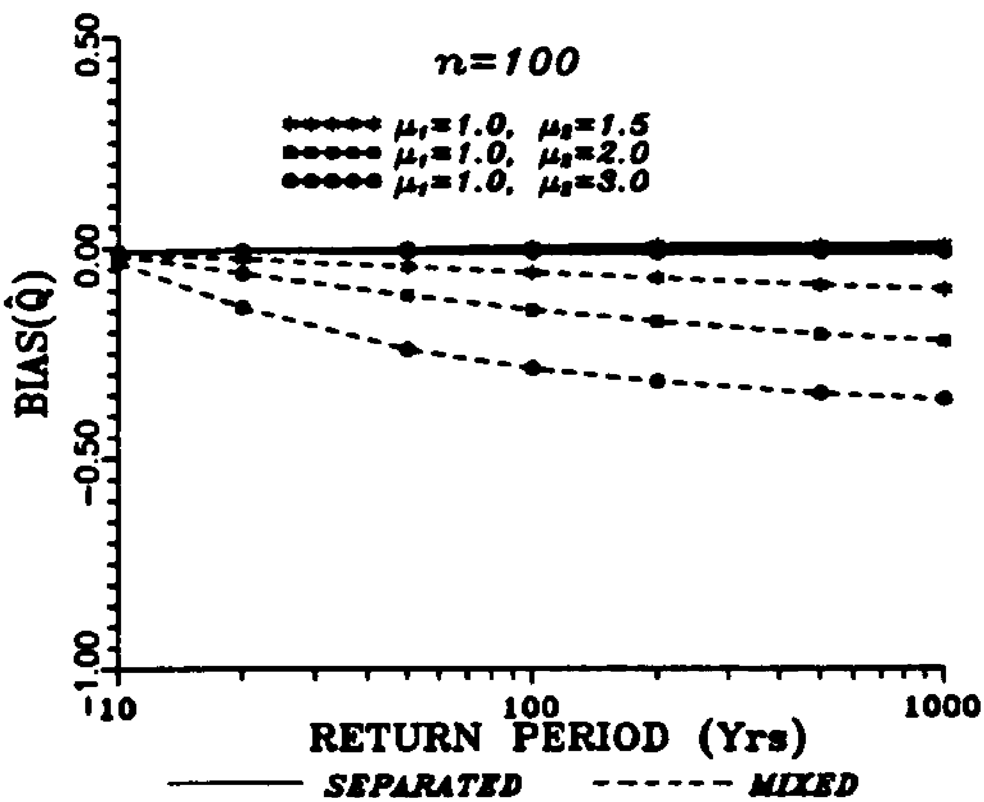
(c)

Figure 5.7 Performances of the Separated and Mixed forms of the Exponential Model for Populations with Different Means: RMSE of Quantiles (a). Sample Size: 50, (b) Sample Size: 100 and (c). Sample Size: 1000



n=50
μ_1=1.0, μ_2=1.5
μ_1=1.0, μ_2=2.0
μ_1=1.0, μ_2=3.0
BIAS($\hat{Q}$)
0.50
0.00
-0.50
-1.00
10
100
1000
RETURN PERIOD (Yrs)
SEPARATED
MIXED

(a)



n=100
μ_1=1.0, μ_2=1.5
μ_1=1.0, μ_2=2.0
μ_1=1.0, μ_2=3.0
BIAS($\hat{Q}$)
0.50
0.00
-0.50
-1.00
10
100
1000
RETURN PERIOD (Yrs)
SEPARATED
MIXED

(b)

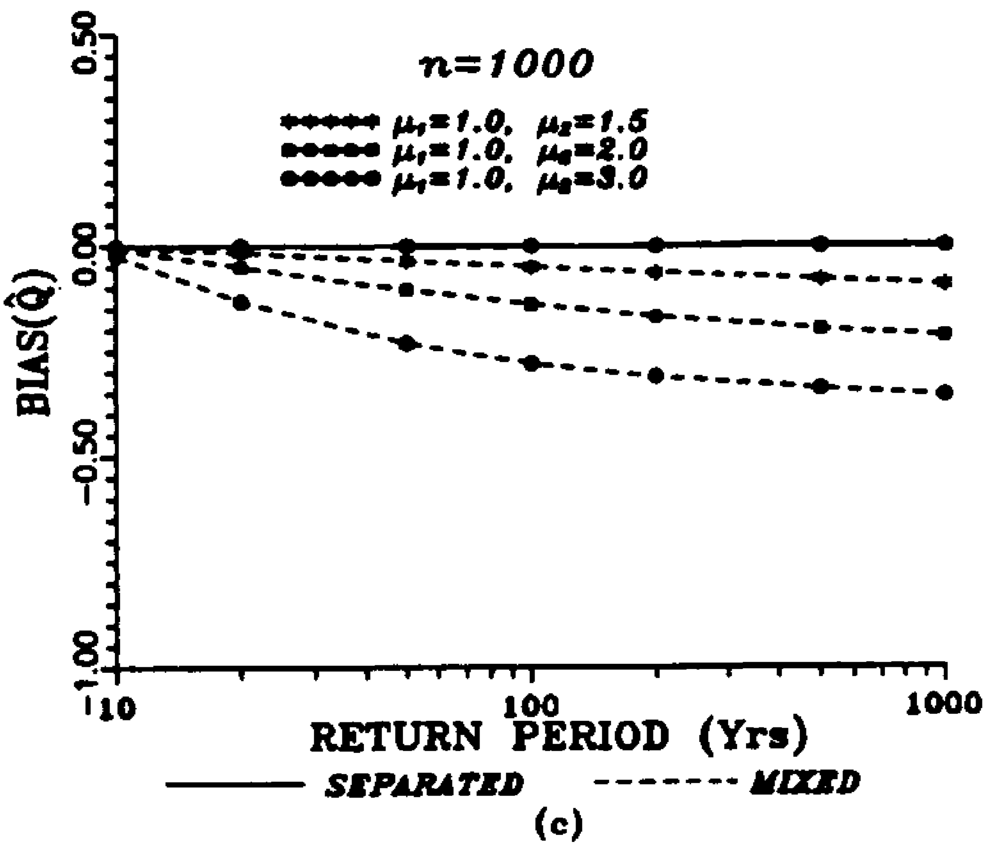


Figure 5.8 Performances of the Separated and Mixed forms of the Exponential Model for Populations with Different Means: BIAS of Quantiles (a). Sample Size: 50, (b) Sample Size: 100 and (c). Sample Size: 1000

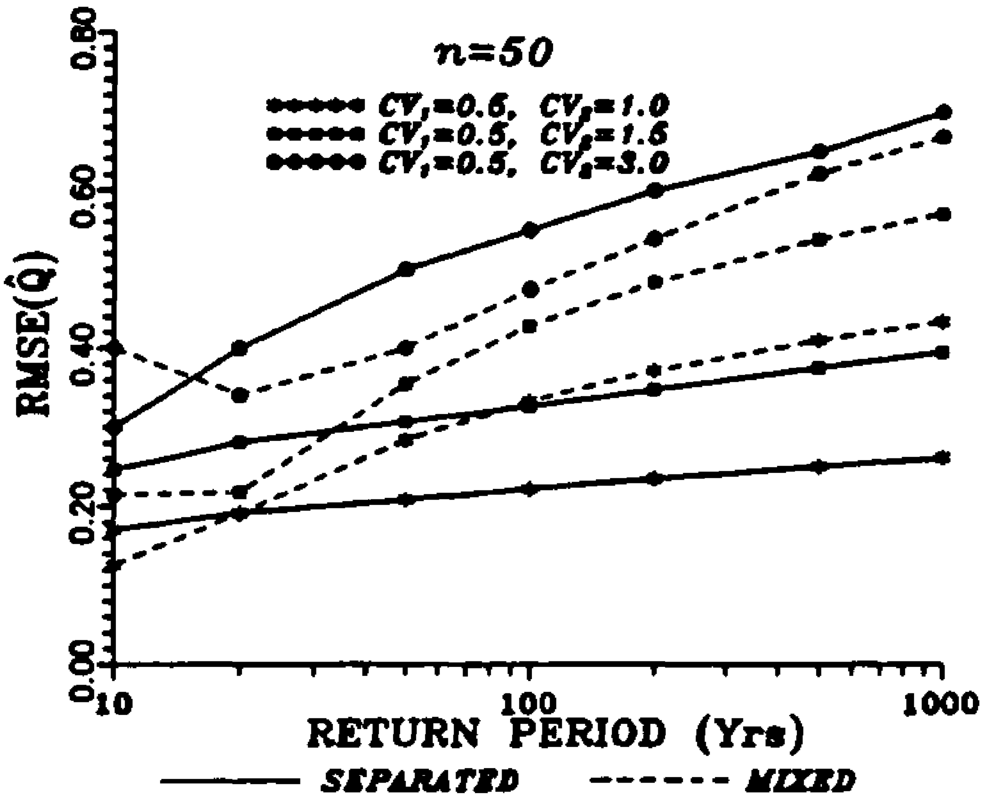
For Weibull Model:

The results of $BIAS$ and $RMSE$ for quantile estimation analysis of nonhomogeneous populations, obtained from the compound Weibull model and the simple Weibull model based on the POME estimator are given in Tables A.91 to A.108 and Tables A.109 to A.126, respectively. These results were used to evaluate the comparative performance of the compound Weibull model relative to the simple Weibull model. In analyzing the effect of degree of nonhomogeneity between the sub-populations on the performances of the compound and simple models in terms of CV's of the sub-populations, the ratio of CV's (CV_2/CV_1) was allowed to vary from 2.0 to 6.0 by keeping the ratio of means constant (say, $\mu_2/\mu_1 = 2$).

As shown from Figure 5.9, when $CV_2/CV_1 \leq 3.0$ the compound model consistently shows superior performance relative to the simple model in terms of $RMSE$ of all quantile estimation and sample sizes tested. Also it appears that when the sample size increases, the compound model continues to improve its performance for over 25 yr return periods. In the case of large ratio of CV ($CV_2/CV_1 = 6.0$) and for small sample sizes (50 or even up to 100), the superior performance of the compound model seems to be negatively influenced by the selected method of parameter estimation, in this case the POME. However, as the sample size increases ($n \geq 100$), this negative influence of the POME estimator appears to be phasing out. In fact, at sample size of 100 the compound model regains its superiority over the simple model for return periods of over 100 years.

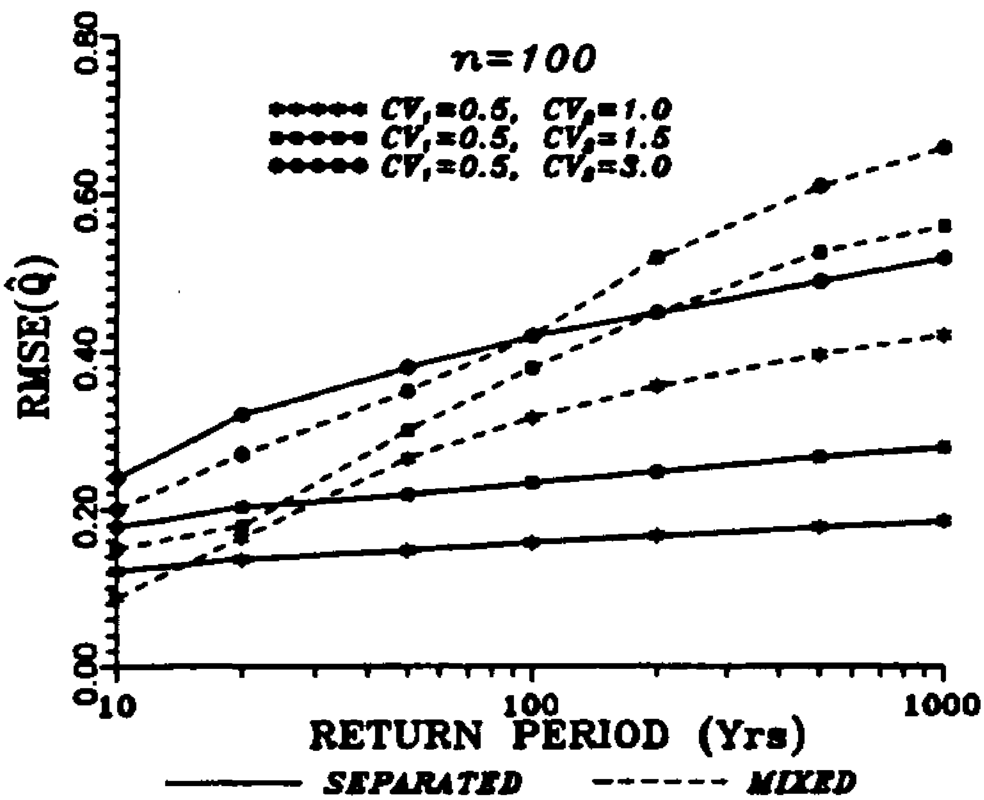
As seen from Figure 5.10, the ratio of $CV_2/CV_1 \leq 3.0$ and for all sample sizes tested, the simple model consistently shows larger negative $BIAS$ of all quantile estimation relative to the compound model. This negative $BIAS$ rapidly increases as the the ratio of CV_2/CV_1 of the sub-populations increases. However, in the case of $CV_2/CV_1 = 6.0$, the simple model shows a decreasing trend in negative $BIAS$ towards the lower return periods, may be as a result of the POME estimator.

Here again the results show that, as in the case of the exponential model, the simple formulation of the Weibull model under-estimates the quantile values for mixed populations where two or more sub-populations with different variances are mixed. The under-estimation of quantile values increases further as the degree of nonhomogeneity (in terms of variances) of sub-populations increases. These results suggest that to describe mixed flood populations, the compound formulations would be the most feasible alternative to the simple formulations, which are widely used in traditional stochastic flood frequency models.



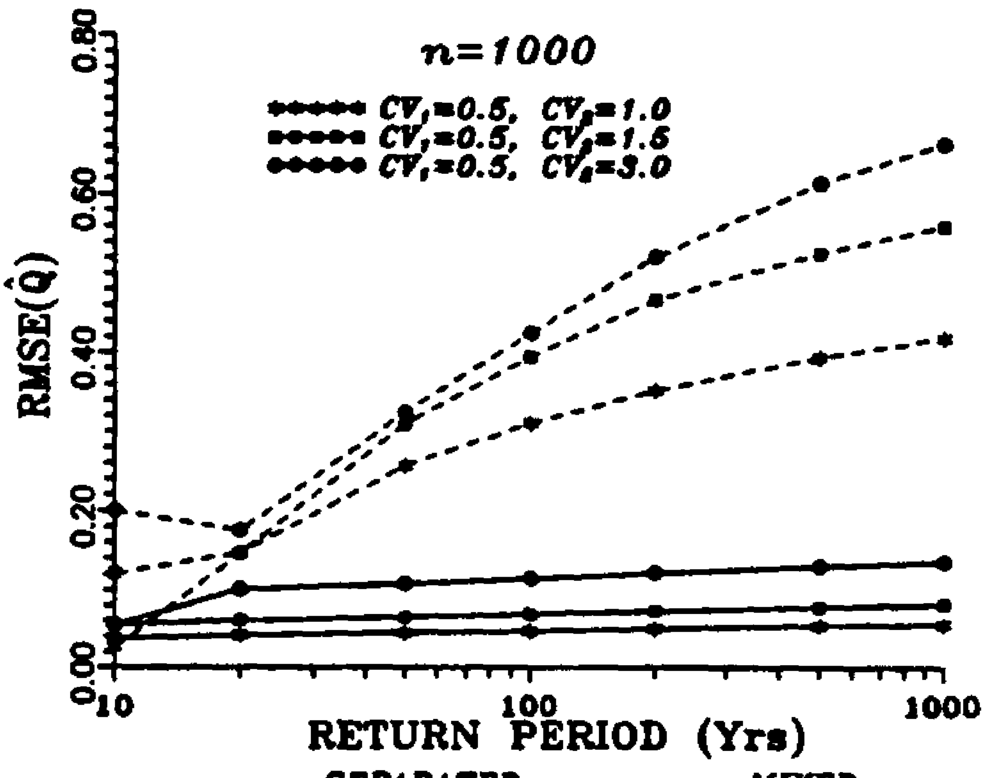
n=50
CV1=0.5, CV2=1.0
CV1=0.5, CV2=1.5
CV1=0.5, CV2=3.0
RMSE(Q̂)
0.00
0.20
0.40
0.60
0.80
10
100
1000
RETURN PERIOD (Yrs)
SEPARATED
MIXED

(a)



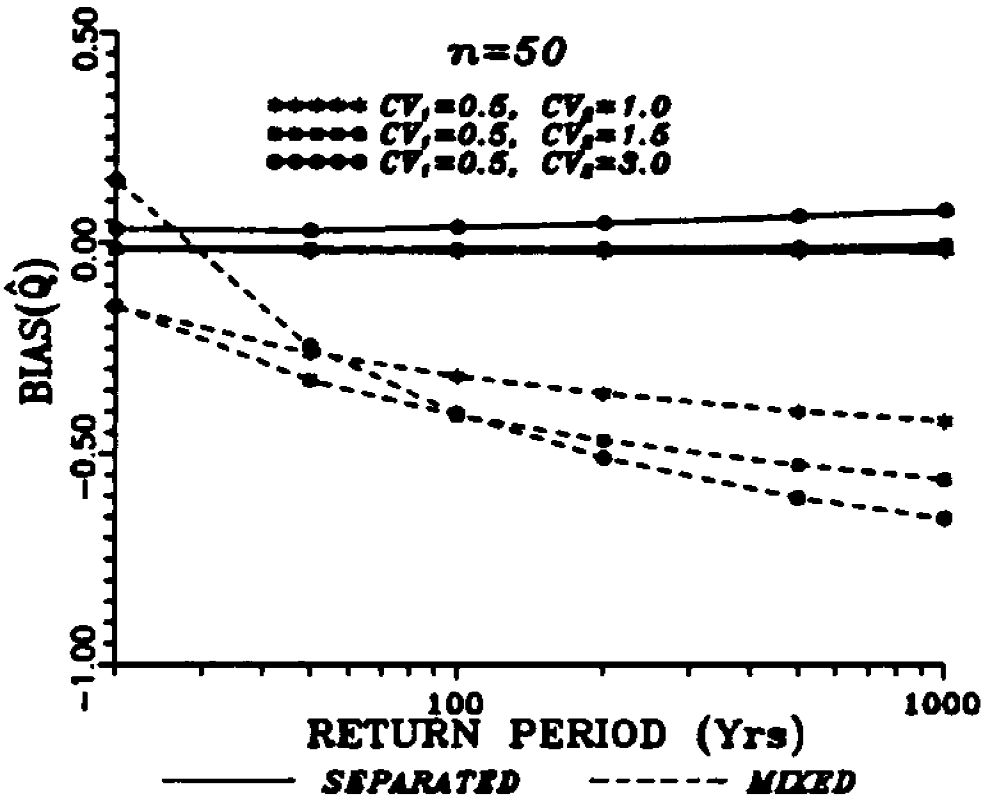
n=100
CV1=0.5, CV2=1.0
CV1=0.5, CV2=1.5
CV1=0.5, CV2=3.0
RMSE(Q̂)
0.00
0.20
0.40
0.60
0.80
10
100
1000
RETURN PERIOD (Yrs)
SEPARATED
MIXED

(b)



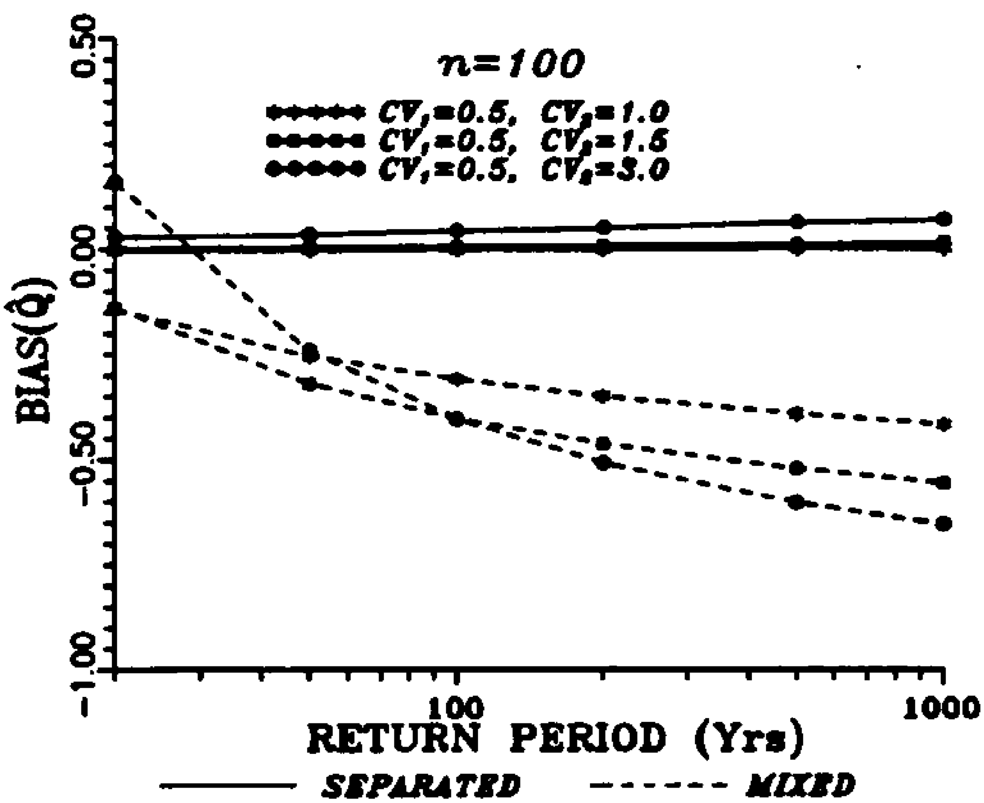
(c)

Figure 5.9 Performances of the Separated and Mixed forms of the Weibull Model for Populations with Same Means but Different CV's: RMSE of Quantiles (a). Sample Size: 50, (b) Sample Size: 100 and (c). Sample Size: 1000



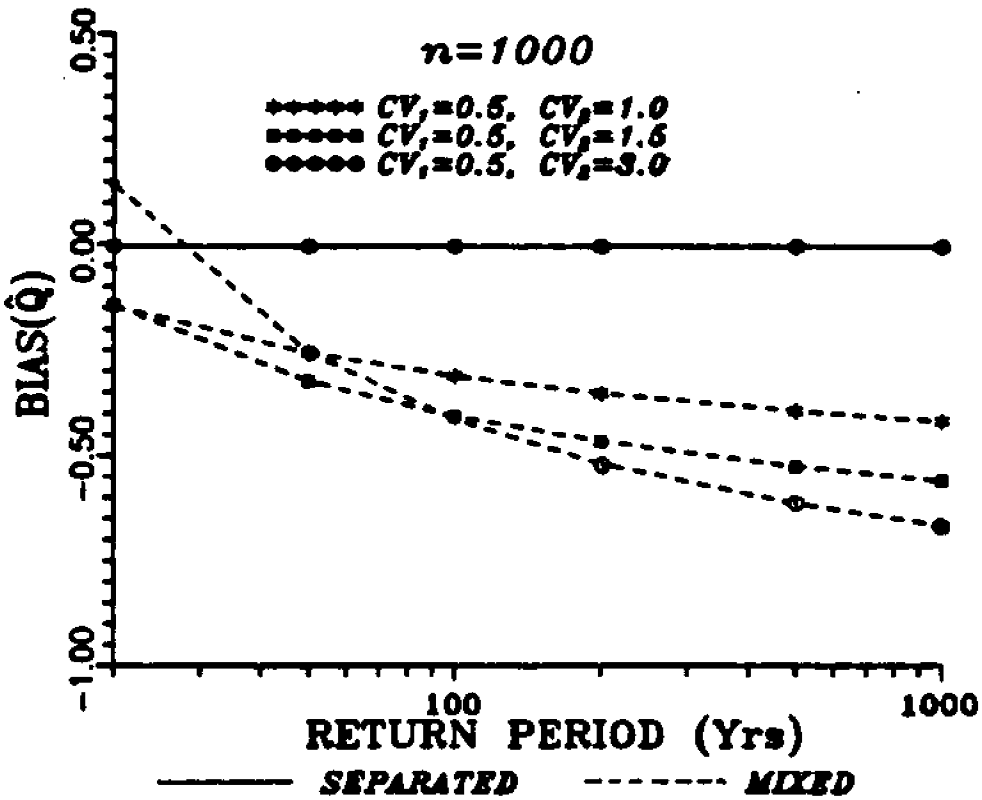
n=50
CV1=0.5, CV2=1.0
CV1=0.5, CV2=1.5
CV1=0.5, CV2=3.0
0.50
0.00
-0.50
-1.00
BIAS(Q̂)
100
1000
RETURN PERIOD (Yrs)
SEPARATED
MIXED

(a)



n=100
CV1=0.5, CV2=1.0
CV1=0.5, CV2=1.5
CV1=0.5, CV2=3.0
0.50
0.00
-0.50
-1.00
BIAS(Q̂)
100
1000
RETURN PERIOD (Yrs)
SEPARATED
MIXED

(b)



(c)

Figure 5.10 Performances of the Separated and Mixed forms of the Weibull Model for Populations with Same Means but Different CV's: BIAS of Quantiles (a). Sample Size: 50, (b) Sample Size: 100 and (c). Sample Size: 1000

5.3 Analysis of Descriptive Properties

Following the analysis of the asymptotic predictive properties (tasks 1 to 3) of selected flood like probability models, the descriptive properties (tasks 4 and 5) of those models were examined on the observed flood series of Louisiana given in Table B.1. Before examining the descriptive characteristics of those selected flood models as discussed in section 4.4.1 (step *a* to *e*), the observed flood sequences were tested for underlying model assumptions.

5.3.1 Testing of Model Assumptions on Observed Data

Step *a*: Hydroclimatic Separation of Flood Sequences

The original intention of this study was to use a more detailed hydroclimatic separation approach; however, the time consuming nature and the current stage of this approach (Hirschboeck and Cruise, 1990) led to the use of a somewhat simplified hydroclimatic separation approach instead. Therefore, following the discussion in section 4.4.2, on the assumption that the flood generating mechanisms of flood events in Louisiana are seasonally controlled, each observed flood series was separated into two sub-populations as 'winter/spring' and 'summer/fall' .

Step *b*: Poisson Test in Selecting Threshold Level

The observed flood series were tested for the Poisson assumption according to the procedure explained in section 4.4.3 to ensure statistical independence. The results of the Poisson test on annually homogeneous PDS are given in Table B.2. It shows that at $\alpha = 10\%$, 18 of the 27 flood series were accepted by the Poisson hypothesis.

Therefore, for those 18 flood series the corresponding Poisson admissible threshold levels were used to define the PDS's with mutually independent flood events. For those 9 remaining flood series the initial USGS threshold levels could be used to define the PDS's in which flood events may or may not satisfy the mutual independence assumption. However, those 9 flood series were not included in the subsequent analysis of this study. Those selected threshold levels for homogeneous PDS's were also used in the analysis of nonhomogeneous PDS's, utilizing the regenerative property of the Poissonian process (Cunnane, 1979).

Step *c*: Mann-Whitney Test for Mixed Populations

Following the discussion on the reasoning and the procedure of the modified Mann-Whitney U statistical test outlined in section 4.4.4, the hydroclimatically separated sub-populations for each flood series were tested for statistical similarities. The results of this test are given in Table B.3. It shows at $\alpha = 10\%$, statistically distinct sub-populations were found to exist in only 4 of the 18 flood series. Interestingly enough, all of these 4 flood series (Station Code: 12, 14, 15, and 16) were located in the southwestern (SW) hydrologic region of Louisiana (Naghavi et al., 1989).

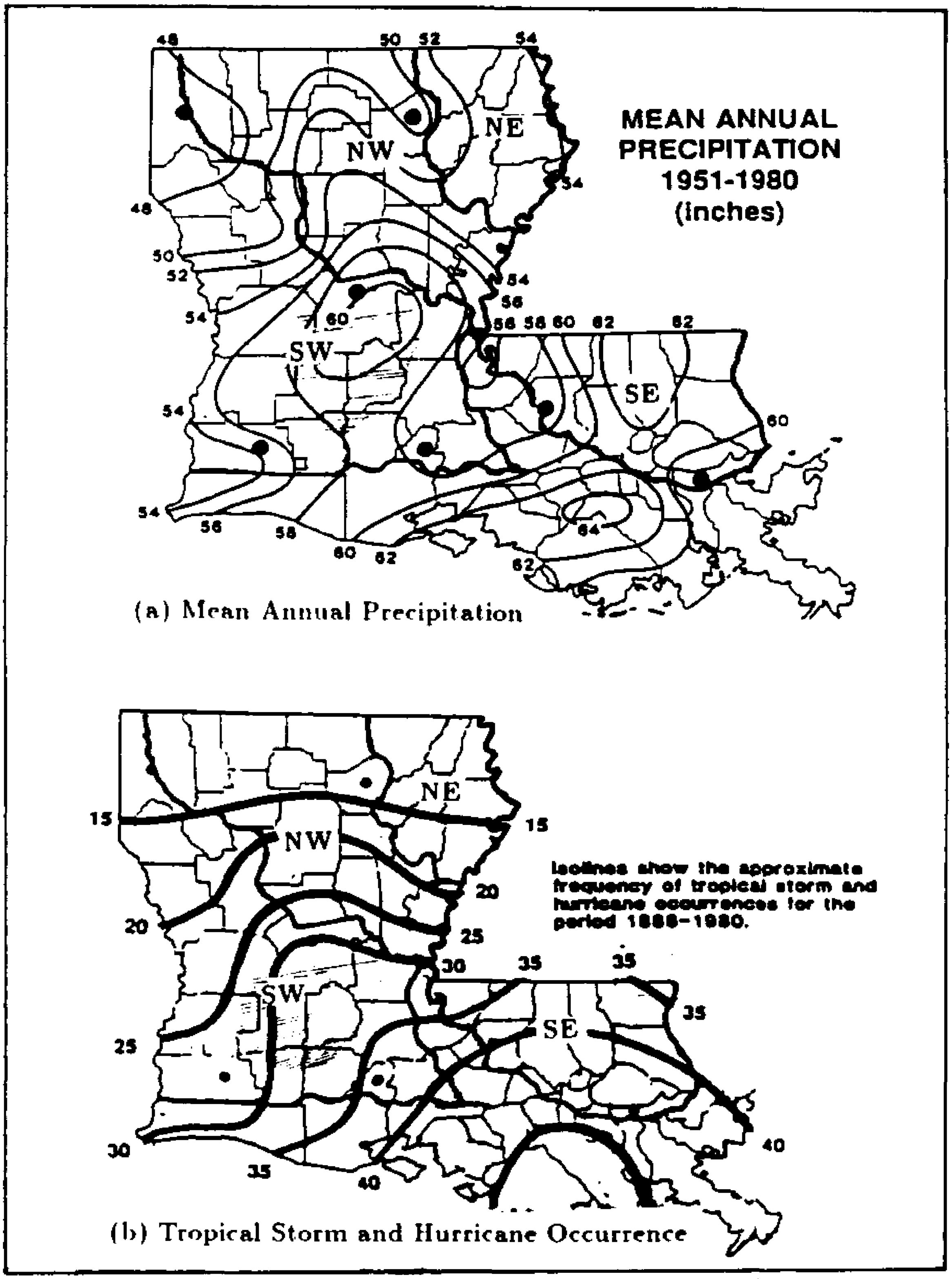
The reason for this unique behaviour of the SW hydrologic region is clearly demonstrated in Figure 5.11. As seen from Figure 5.11(a), the stations which are hydroclimatically sensitive with mixed populations are located in an area of the state, that is unique in its precipitation regime compared to the surrounding areas. The reason for this annual precipitation pattern is partly related to a slight topographic effect in the central part of the state. This annual pattern, is probably also related to the frequency of occurrence of tropical storms and hurricanes in the same area compared to the other areas as shown in Figure 5.11(b).

Step *d*: Exponential Test For Marginal Distribution

Before the exponential model was fitted, the annually homogeneous observed flood series were tested for the exponential hypothesis using two EDF tests described in section 4.4.5. The results of these tests are given in Table B.4. It shows that at $\alpha = 5\%$, 6 of the 18 flood series were accepted by both the EDF tests for their corresponding Poisson threshold levels. Flood series at 6 other stations were also accepted by both the EDF tests after raising those respective threshold levels beyond the Poisson admissible threshold levels. The remaining flood series were rejected by both the EDF tests. These results highlight the relative inflexibility of the exponential marginal in fitting to observed flood data and thereby, justify the use of a more flexible marginal, instead.

Step *e*: Feasibility of Weibull Marginal for Observed Data

On the basis of those observed flood series, a feasibility of the Weibull distribution as a marginal for the selected stochastic flood model was discussed in section 4.3.1 (task 1). As seen from Figure 4.1, the traditional exponential distribution ($a = 1$) has failed to describe the behaviour of most of the observed flood series in Louisiana. In contrast, the Weibull distribution with $a < 1.0$ was very successful in describing 15 of the 18 observed flood series, as the estimated points (CV vs a) of those 15 flood series are shown to follow the theoretical relationship described by Eqn. (3.24).



(a) Mean Annual Precipitation

(b) Tropical Storm and Hurricane Occurrence

Figure 5.11: Some Precipitation Patterns in Louisiana
(Hirschboeck and Coxe, 1991)

However, the Weibull distribution has failed to describe the behaviour of the three remaining observed flood series with $CV \geq 1.47$ (Station Code: 12, 13, and 24) as shown by the outlying points on Figure 4.1. From the subsequent analysis of fittings of observed data to the Weibull model, it can be seen from Table B.5, those same three flood series have the largest SRMSE ($\geq .501$) of quantile estimates. Hence, these observations are comparable to the discussion made in section 5.2.1 (task 1) on the limitation of the predictive ability of the Weibull model, when $CV \geq 1.5$.

5.3.2 Task 4: Fitting of Exponential Vs. Weibull Models

In analyzing the descriptive properties, the simple and compound Weibull models were fitted to observed flood series at the Poisson admissible threshold levels, while the exponential admissible threshold levels were used for exponential models.

For Simple Formulation:

The results of the analysis of descriptive properties for both simple formulations (exponential and Weibull) based on annually homogeneous data are given in Table B.5. It shows that the simple Weibull model always resulted in better fits in terms of SRMSE, relative to the exponential counterpart. This is due to the additional flexibility introduced by the shape parameter a of the Weibull model when fitting to the observed flood series. It also shows that in 5 cases (Station Code: 2, 13, 14, 19, and 23), even through the marginal distributions were accepted as exponentially distributed, the Weibull model resulted in significantly better fits. This added superiority of the Weibull model is due to the increase of flexibility in quantile estimation caused by the additional shape parameter.

For Compound Formulation:

The results of the fittings of the compound formulations (exponential and Weibull) to the annually nonhomogeneous observed data are given in Table B.6. The results show that the compound Weibull model did not always result in better fits to the observed data compared to the exponential counterpart, as it did in the case of the simple formulation. In 3 cases (Station Code: 5, 18, and 19), the compound exponential model resulted in better fits to the observed data than did the Weibull counterpart. The above mixed results of the compound formulations were caused by the errors made during: (*a*) hydroclimatic separation, since in most of the flood series distinct sub-populations were not shown to exist, and (*b*) parameter estimation, as seen from the results of the compound Weibull model with six parameters which introduces relatively larger errors compared to the three parameter simple Weibull model than that of the exponential counterpart.

It is also noted that the compound Weibull model resulted in an overall better fit to the observed data than the compound exponential model in terms of overall average SRMSE. On the basis of the exponentially accepted 12 flood series, the overall average SRMSE for the compound Weibull model was 0.188 and for the compound exponential model was 0.231.

5.3.3 Task 5: Fitting of Simple Vs. Compound Forms

When comparing the results of the fittings of both simple formulations (Table B.5) to that of the respective compound formulations (Table B.6) the observations made are summarized in Table 5.2.

Table 5.2 Comparison of Overall Fittings of Simple and Compound Forms

Hydrologic Regions of LA	Exponential Model			Weibull Model		
	Average SRMSE		$(\frac{S-C}{C})$ in %	Average SRMSE		$(\frac{S-C}{C})$ in %
	Simple	Compound		Simple	Compound	
SE	0.215	0.210	+ 2.5	0.177	0.182	- 2.0
SW	0.270	0.236	+ 14.3	0.273	0.251	+ 8.8
NW	0.314	0.314	0.0	0.283	0.281	+ 0.5

where

S = overall average SRMSE of simple model,

C = overall average SRMSE of compound model, and

$(\frac{S-C}{C})$ = % reduction of overall average SRMSE w.r.t the compound model.

It shows that in SW hydrologic region of Louisiana, both the exponential and Weibull compound models resulted in significantly superior fits to the observed data in terms of % reduction of overall average SRMSE than that of the respective simple models. This unique behaviour in SW region of Louisiana was already observed and discussed in section 5.3.1 (step c).

On the basis of results of both simple and compound formulations of the exponential and Weibull models, the fittings of three representative flood series from SE, NW and SW hydrologic regions of Louisiana are shown in Figure 5.12, 5.13, and

5.14, respectively. Those figures further reveal the importance of the compound formulations for regions like SW of Louisiana.

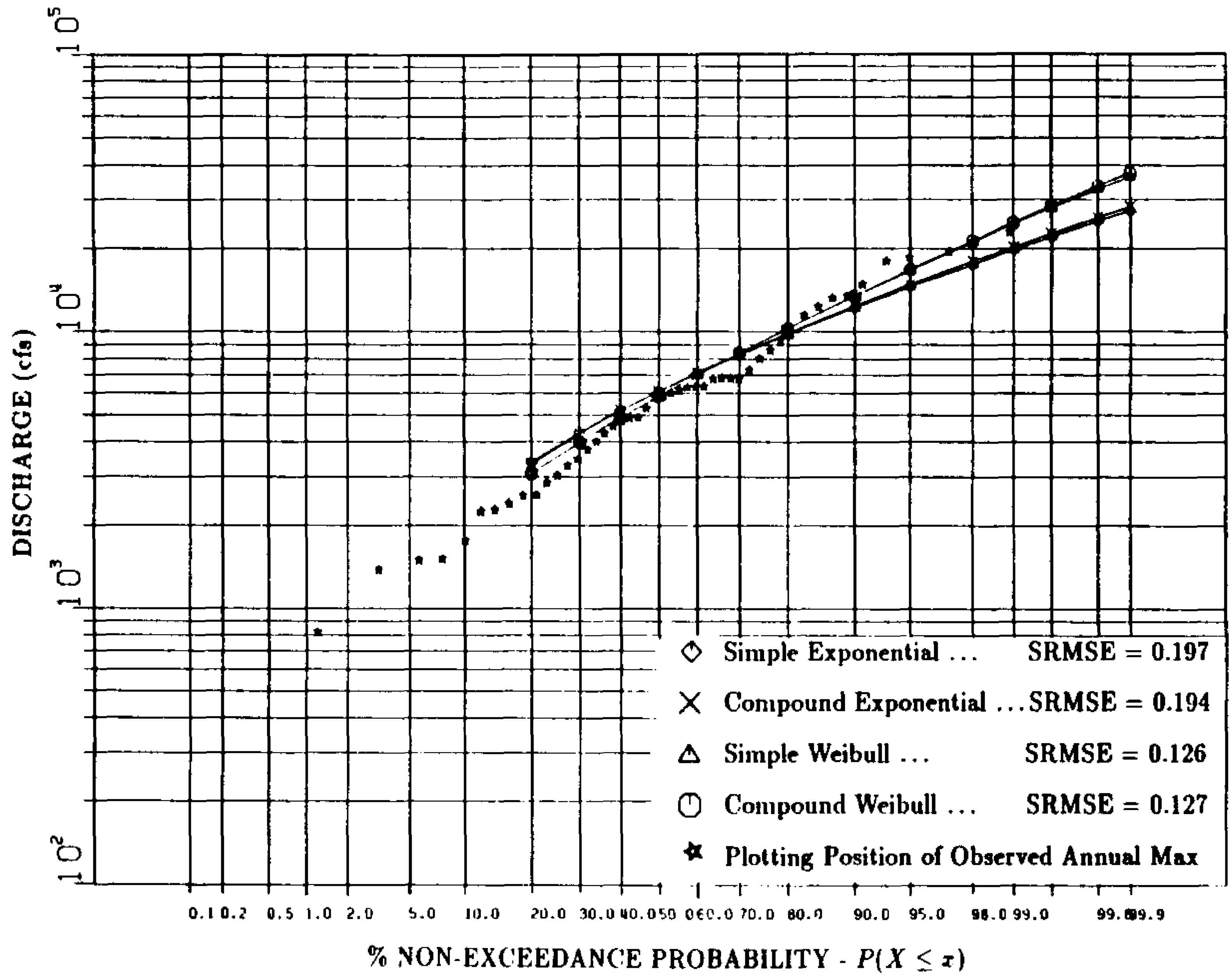


Figure 5.12: Fit of Poisson Partial Duration Models to Observed PDS: Station No. 2 in SE Region of LA

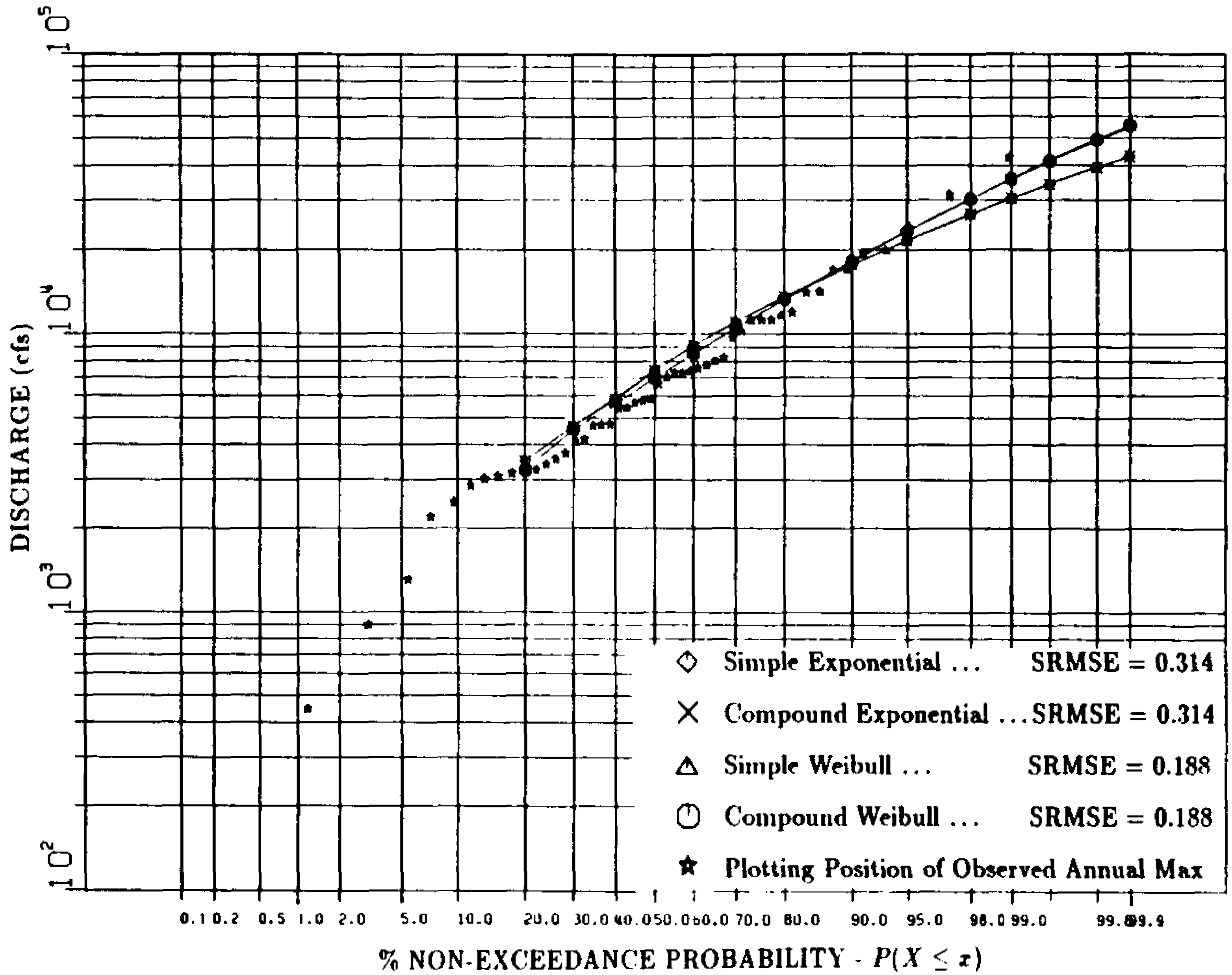


Figure 5.13: Fit of Poisson Partial Duration Models to Observed PDS: Station No. 23 in NW Region of LA

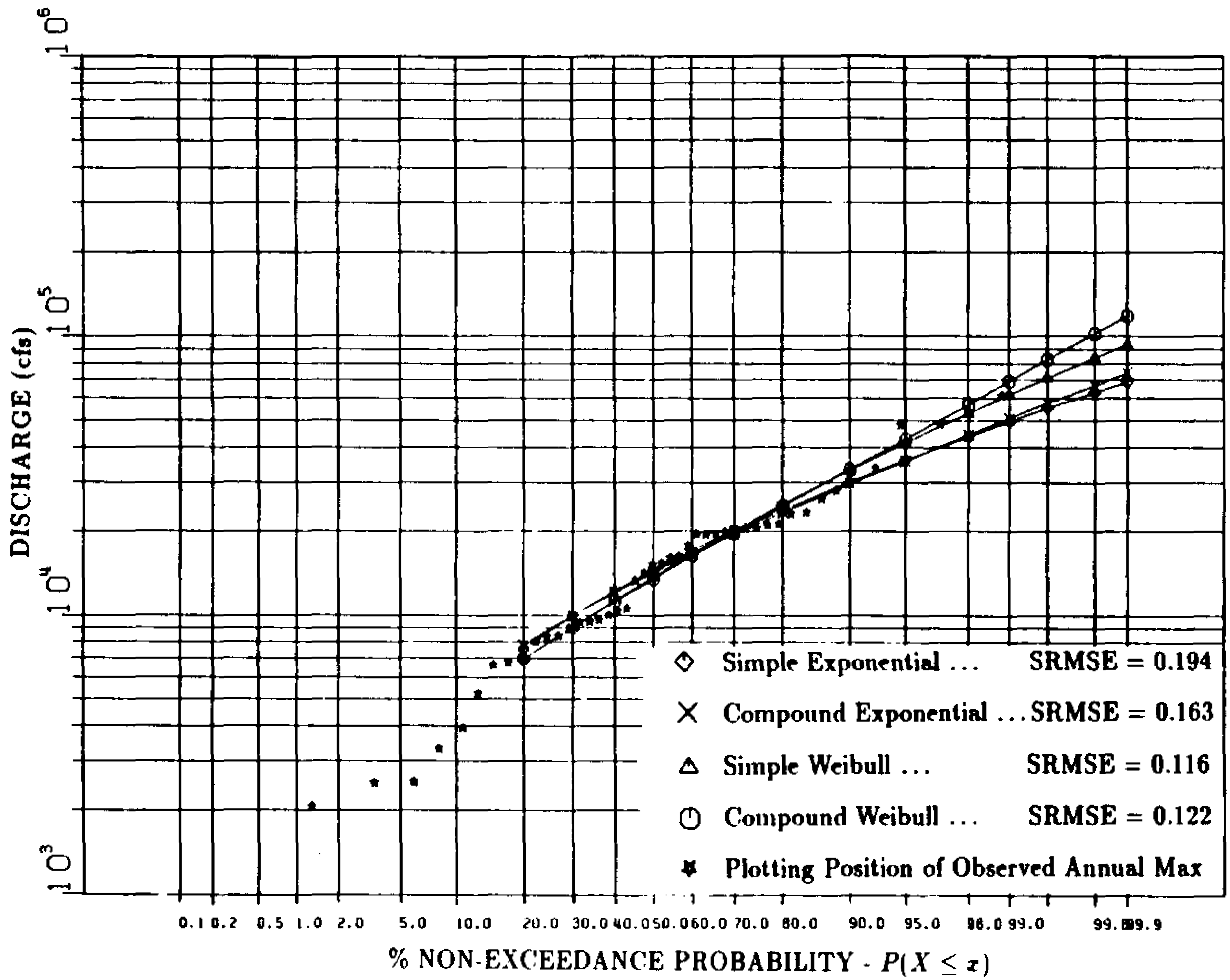


Figure 5.14: Fit of Poisson Partial Duration Models to Observed PDS: Station No. 14 in SW Region of LA

Chapter 6

Summary and Conclusion

This study was concerned with evaluating the statistical performance of various formulations of the exact extreme value Poisson partial duration model. Hence, the objective was to decide which formulation was suitable if mixed climatological processes were present in the observed flood series of Louisiana. The study reveals the following observations, in terms of both predictive and descriptive properties of the various model formulations tested;

a. A more flexible form, the Weibull model performs well over the the traditional exponential model for specific population conditions which are shown to exist in most of the observed flood series of Louisiana,

b. The POME as a Weibull quantile estimator demonstrates advantages over the PWM in most of the population cases tested, and

c. The compound formulations of the both Weibull and exponential models show superior performances over the respective simple formulations if statistically distinct mixed populations are present in the observed flood series.

Initially, the relative performance of the Weibull model was compared to the traditional exponential model. Both models were compared in terms of $BIAS$ and

$RMSE$ of quantile estimation over a variety of population cases.

The results of the performance analysis in terms of $RMSE$ suggest that for $CV \leq 0.5$, the Weibull model consistently performs well over the exponential counterpart, even with samples containing 1 flood event per year ($\lambda = 1$). For samples having $0.5 \leq CV \leq 1.0$, the Weibull model continues its superior performance over the exponential model, at this time only when those samples contain at least 2 flood events per year ($\lambda = 2$).

According to the data shown in Tables B.2 and B.3, most of the observed flood series in Louisiana demonstrate an average rate of 1.7 flood events per year and relatively small values of CV, at Poisson admissible threshold levels. Hence, the Weibull model would be recommended as a better alternative to the exponential model for most observed flood series in Louisiana, particularly for those series which satisfy the Poisson process.

The results of the performance analysis further suggest that for samples having $1.0 \leq CV \leq 1.5$, for the Weibull model to be preferable over the exponential model, those samples should contain at least 3 flood events per year ($\lambda = 3$). In working with observed flood series of Louisiana, it may be possible to have on the average 3 floods events per year, only at the initial USGS threshold levels. In the case of $1.5 \leq CV \leq 3.0$, the results show that the Weibull model completely deteriorates with its asymptotic predictive power over the exponential model. However, the use of the exponential model is too not appropriate for flood series with large CV's. Therefore, further research is needed in the area of selecting marginal distributions for flood exceedances, particularly when large variance is expected in observed flood series.

The results of the performance analysis in terms of $BIAS$ demonstrate that for small samples with large variability ($1.5 \leq CV \leq 3.0$), the Weibull model over estimates the quantile values compared to the exponential model. However, for small samples with small variability ($0.5 \leq CV < 1.5$), although the Weibull model under estimates the quantile values the results could be considered as comparable to that of the exponential model.

Next, a brief analysis of the comparative performance of the POME and PWM for estimating quantiles of the Weibull model was performed, following the study by Singh et al. (1990). The estimators were compared in terms of $BIAS$ and $RMSE$ of quantile estimation for a wide range of population variation. The results show that for most population cases tested, the POME consistently performs well over the PWM. However, for small samples with large CV's, the PWM estimator appears to be superior in estimating Weibull quantiles at small return periods. Following those results, the POME would be recommended as a feasible parameter estimation method for the Weibull model when fitting to observed flood series where they show relatively small values of CV's. In addition, the PWM would be suggested as a feasible regional Weibull estimator, where large variation is expected in the regional observed flood data.

Finally, the relative performance of compound formulations of both the Weibull and exponential models were compared to the respective simple counterparts. Both formulations of the exponential model were compared in terms of $BIAS$ and $RMSE$ of quantile estimations for a wide variation of population means. The results of the performance analysis in terms of $RMSE$ suggest that for increasing sample size and ratio of the population means, the compound model demonstrates its superiority over the simple model.

Similarly, both formulations of the Weibull model were also compared in terms of $BIAS$ and $RMSE$ of quantile estimation based on the POME estimator for wide variation of population CV's, while keeping population means constant. As before, in terms of $RMSE$ the compound model continuous to perform well for increasing sample size and the ratio of population CV's. When the ratio of CV's becomes very large, for small samples the superior performance of the compound model was influenced by the POME estimator. However, as the sample size increases, the compound model again shows superior performance over the simple model.

The results of the performance analysis in terms of $BIAS$ of both the exponential and Weibull models suggest that the simple model formulations under-estimate the quantile values for mixed populations. The under-estimation of quantile values rapidly increases as the degree of nonhomogeneity of mixed populations increases. These results (in terms of both $BIAS$ and $RMSE$) suggest that when the flood events from different populations are mixed, the compound model formulation would be a feasible approach to describe mixed flood populations.

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Appendix A

Analysis of Simulated Data

SIMPLE POISSON PARTIAL DURATION MODEL WITH EXPONENTIAL DISTRIBUTION AS MARGINAL

TABLE A.1 BIAS OF SELECTED QUANTILES : (M = 1.0 L = 1.0)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	0.367	1.500	2.250	2.970	3.902	4.600	5.296	6.214	6.907
10		-0.130	-0.025	-0.013	-0.008	-0.004	-0.002	0.000	0.001	0.002
20		-0.075	-0.012	-0.005	-0.002	0.000	0.001	0.002	0.003	0.004
30		-0.041	-0.008	-0.004	-0.002	-0.001	0.000	0.000	0.000	0.001
50		-0.028	-0.011	-0.009	-0.008	-0.007	-0.007	-0.006	-0.006	-0.006
100		-0.021	-0.005	-0.004	-0.003	-0.002	-0.002	-0.002	-0.001	-0.001
200		0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
500		-0.002	0.000	0.001	0.001	0.001	0.001	0.001	0.001	0.001
1000		-0.006	-0.002	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001

TABLE A.2 RMSE OF SELECTED QUANTILES : (M = 1.0 L = 1.0)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	0.367	1.500	2.250	2.970	3.902	4.600	5.296	6.214	6.907
10		1.049	0.388	0.351	0.339	0.333	0.331	0.330	0.329	0.329
20		0.679	0.270	0.246	0.238	0.234	0.233	0.232	0.231	0.231
30		0.552	0.225	0.205	0.198	0.195	0.193	0.192	0.192	0.191
50		0.405	0.166	0.151	0.146	0.143	0.142	0.141	0.141	0.140
100		0.286	0.120	0.109	0.105	0.103	0.102	0.102	0.101	0.101
200		0.211	0.086	0.078	0.075	0.074	0.073	0.073	0.072	0.072
500		0.132	0.054	0.049	0.047	0.046	0.045	0.045	0.045	0.045
1000		0.089	0.038	0.035	0.033	0.033	0.032	0.032	0.032	0.032

SIMPLE POISSON PARTIAL DURATION MODEL WITH EXPONENTIAL DISTRIBUTION AS MARGINAL

TABLE A.3 BIAS OF SELECTED QUANTILES : (M = 1.0 L = 2.0)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	1.060	2.193	2.944	3.663	4.595	5.293	5.989	6.907	7.600
10		-0.017	-0.004	-0.001	0.001	0.003	0.003	0.004	0.005	0.005
20		-0.008	-0.003	-0.002	-0.002	-0.001	-0.001	-0.001	0.000	0.000
30		-0.009	-0.007	-0.006	-0.006	-0.006	-0.006	-0.005	-0.005	-0.005
50		0.002	0.003	0.004	0.004	0.004	0.004	0.005	0.005	0.005
100		-0.007	-0.004	-0.004	-0.003	-0.003	-0.003	-0.003	-0.002	-0.002
200		-0.003	-0.002	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001
500		-0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1000		-0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

TABLE A.4 RMSE OF SELECTED QUANTILES : (M = 1.0 L = 2.0)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	1.060	2.193	2.944	3.663	4.595	5.293	5.989	6.907	7.600
10		0.315	0.246	0.236	0.231	0.228	0.227	0.226	0.226	0.226
20		0.224	0.179	0.172	0.169	0.167	0.166	0.165	0.165	0.165
30		0.176	0.142	0.137	0.134	0.133	0.132	0.132	0.131	0.131
50		0.141	0.113	0.108	0.106	0.105	0.104	0.104	0.103	0.103
100		0.096	0.077	0.074	0.073	0.072	0.072	0.072	0.071	0.071
200		0.068	0.055	0.053	0.052	0.051	0.051	0.051	0.051	0.051
500		0.043	0.033	0.032	0.031	0.030	0.030	0.030	0.030	0.030
1000		0.031	0.025	0.023	0.023	0.023	0.022	0.022	0.022	0.022

SIMPLE POISSON PARTIAL DURATION MODEL WITH EXPONENTIAL DISTRIBUTION AS MARGINAL

TABLE A.5 BIAS OF SELECTED QUANTILES : (M = 1.0 L = 3.0)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	1.465	2.599	3.349	4.069	5.001	5.699	6.394	7.312	8.006
10		-0.005	0.000	0.001	0.002	0.002	0.003	0.003	0.003	0.004
20		-0.017	-0.013	-0.012	-0.011	-0.010	-0.010	-0.010	-0.009	-0.009
30		-0.004	-0.004	-0.004	-0.004	-0.004	-0.004	-0.004	-0.004	-0.004
50		-0.003	-0.001	-0.001	0.000	0.000	0.000	0.000	0.000	0.000
100		-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001
200		0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
500		-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001
1000		0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001

TABLE A.6 RMSE OF SELECTED QUANTILES : (M = 1.0 L = 3.0)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	1.465	2.599	3.349	4.069	5.001	5.699	6.394	7.312	8.006
10		0.222	0.194	0.188	0.186	0.184	0.183	0.182	0.182	0.181
20		0.158	0.139	0.136	0.134	0.133	0.132	0.132	0.132	0.131
30		0.130	0.115	0.112	0.111	0.110	0.109	0.109	0.108	0.108
50		0.101	0.089	0.086	0.085	0.084	0.084	0.084	0.083	0.083
100		0.071	0.063	0.061	0.060	0.060	0.059	0.059	0.059	0.059
200		0.050	0.044	0.043	0.042	0.042	0.041	0.041	0.041	0.041
500		0.032	0.028	0.027	0.027	0.027	0.027	0.026	0.026	0.026
1000		0.021	0.019	0.018	0.018	0.018	0.018	0.018	0.018	0.018

SIMPLE POISSON PARTIAL DURATION MODEL WITH WEIBULL DISTRIBUTION AS MARGINAL (BY POME)

TABLE A.7 BIAS OF SELECTED QUANTILES : (M = 1.0 CV = 0.5 L = 1.0)

SAMPLE SIZE	T = 2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q = 0.700	1.369	1.661	1.895	2.158	2.334	2.496	2.693	2.832
10	-0.101	-0.036	-0.038	-0.042	-0.045	-0.046	-0.048	-0.049	-0.049
20	-0.076	-0.019	-0.020	-0.021	-0.023	-0.023	-0.024	-0.024	-0.024
30	-0.053	-0.012	-0.013	-0.014	-0.015	-0.015	-0.016	-0.016	-0.016
50	-0.030	-0.010	-0.011	-0.012	-0.012	-0.013	-0.013	-0.013	-0.013
100	-0.015	-0.005	-0.005	-0.006	-0.006	-0.006	-0.006	-0.007	-0.007
200	-0.002	-0.001	-0.002	-0.003	-0.003	-0.003	-0.003	-0.004	-0.004
500	-0.001	0.000	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001
1000	-0.003	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001

TABLE A.8 RMSE OF SELECTED QUANTILES : (M = 1.0 CV = 0.5 L = 1.0)

SAMPLE SIZE	T = 2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q = 0.700	1.369	1.661	1.895	2.158	2.334	2.496	2.693	2.832
10	0.511	0.199	0.182	0.191	0.209	0.223	0.236	0.254	0.266
20	0.405	0.134	0.126	0.131	0.143	0.152	0.161	0.172	0.180
30	0.327	0.108	0.103	0.107	0.115	0.121	0.128	0.135	0.141
50	0.241	0.081	0.074	0.077	0.083	0.088	0.093	0.099	0.103
100	0.154	0.058	0.056	0.058	0.064	0.067	0.071	0.076	0.079
200	0.109	0.041	0.038	0.040	0.043	0.046	0.048	0.051	0.053
500	0.069	0.026	0.024	0.025	0.027	0.028	0.030	0.032	0.033
1000	0.046	0.018	0.017	0.018	0.019	0.020	0.021	0.022	0.023

SIMPLE POISSON PARTIAL DURATION MODEL WITH WEIBULL DISTRIBUTION AS MARGINAL (BY POME)

TABLE A.9 BIAS OF SELECTED QUANTILES : (M = 1.0 CV = 0.5 L = 2.0)

SAMPLE SIZE	T = 2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q = 1.161	1.641	1.887	2.094	2.333	2.495	2.646	2.832	2.964
10	-0.017	-0.018	-0.020	-0.021	-0.022	-0.022	-0.023	-0.023	-0.023
20	-0.008	-0.011	-0.013	-0.014	-0.015	-0.016	-0.017	-0.017	-0.017
30	-0.008	-0.009	-0.009	-0.010	-0.010	-0.010	-0.010	-0.010	-0.010
50	-0.002	-0.001	0.000	0.000	0.000	0.000	0.000	0.001	0.001
100	-0.004	-0.004	-0.004	-0.004	-0.004	-0.004	-0.004	-0.004	-0.004
200	-0.002	-0.002	-0.002	-0.002	-0.002	-0.002	-0.002	-0.002	-0.002
500	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1000	-0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

TABLE A.10 RMSE OF SELECTED QUANTILES : (M = 1.0 CV = 0.5 L = 2.0)

SAMPLE SIZE	T = 2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q = 1.161	1.641	1.887	2.094	2.333	2.495	2.646	2.832	2.964
10	0.158	0.124	0.127	0.135	0.146	0.154	0.161	0.171	0.177
20	0.110	0.089	0.093	0.099	0.107	0.113	0.118	0.125	0.130
30	0.087	0.070	0.073	0.078	0.084	0.089	0.093	0.098	0.102
50	0.068	0.056	0.058	0.062	0.067	0.070	0.074	0.078	0.081
100	0.047	0.038	0.039	0.042	0.045	0.048	0.050	0.053	0.055
200	0.033	0.027	0.029	0.030	0.033	0.035	0.036	0.038	0.040
500	0.021	0.016	0.017	0.018	0.020	0.021	0.022	0.023	0.024
1000	0.015	0.012	0.013	0.014	0.015	0.016	0.016	0.017	0.018

SIMPLE POISSON PARTIAL DURATION MODEL WITH WEIBULL DISTRIBUTION AS MARGINAL (BY POME)

TABLE A.11 BIAS OF SELECTED QUANTILES : (M = 1.0 CV = 0.5 L = 3.0)

SAMPLE SIZE	T = 2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q = 1.354	1.779	2.007	2.202	2.429	2.585	2.730	2.910	3.038
10	-0.010	-0.013	-0.014	-0.015	-0.016	-0.017	-0.017	-0.018	-0.018
20	-0.012	-0.013	-0.014	-0.014	-0.015	-0.015	-0.016	-0.016	-0.016
30	-0.005	-0.006	-0.007	-0.008	-0.008	-0.008	-0.009	-0.009	-0.009
50	-0.003	-0.003	-0.003	-0.003	-0.003	-0.003	-0.003	-0.003	-0.003
100	-0.001	-0.002	-0.002	-0.002	-0.003	-0.003	-0.003	-0.003	-0.003
200	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
500	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001
1000	0.000	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001

TABLE A.12 RMSE OF SELECTED QUANTILES : (M = 1.0 CV = 0.5 L = 3.0)

SAMPLE SIZE	T = 2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q = 1.354	1.779	2.007	2.202	2.429	2.585	2.730	2.910	3.038
10	0.107	0.102	0.108	0.116	0.125	0.132	0.139	0.147	0.152
20	0.076	0.074	0.079	0.084	0.090	0.095	0.099	0.104	0.108
30	0.062	0.059	0.062	0.066	0.071	0.074	0.077	0.081	0.084
50	0.048	0.046	0.048	0.051	0.055	0.058	0.060	0.064	0.066
100	0.034	0.032	0.034	0.036	0.039	0.041	0.043	0.045	0.046
200	0.024	0.022	0.023	0.025	0.027	0.028	0.029	0.031	0.032
500	0.015	0.014	0.015	0.016	0.017	0.018	0.019	0.020	0.020
1000	0.010	0.010	0.011	0.011	0.012	0.013	0.014	0.015	0.015

SIMPLE POISSON PARTIAL DURATION MODEL WITH WEIBULL DISTRIBUTION AS MARGINAL (BYPOME)

TABLE A.13 BIAS OF SELECTED QUANTILES : (M = 1.0 CV = 1.0 L = 1.0)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	0.367	1.500	2.250	2.970	3.902	4.600	5.296	6.214	6.907
10		0.082	-0.031	-0.042	-0.045	-0.044	-0.040	-0.035	-0.028	-0.021
20		0.024	-0.018	-0.024	-0.025	-0.024	-0.022	-0.020	-0.017	-0.014
30		0.009	-0.012	-0.015	-0.016	-0.016	-0.015	-0.014	-0.013	-0.011
50		0.004	-0.014	-0.017	-0.018	-0.018	-0.018	-0.018	-0.017	-0.016
100		-0.004	-0.006	-0.007	-0.008	-0.008	-0.008	-0.008	-0.007	-0.007
200		0.010	-0.001	-0.002	-0.003	-0.004	-0.004	-0.005	-0.005	-0.005
500		0.002	0.000	0.000	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001
1000		-0.004	-0.002	-0.002	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001

TABLE A.14 RMSE OF SELECTED QUANTILES : (M = 1.0 CV = 1.0 L = 1.0)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	0.367	1.500	2.250	2.970	3.902	4.600	5.296	6.214	6.907
10		0.863	0.379	0.364	0.391	0.448	0.502	0.561	0.646	0.715
20		0.654	0.270	0.259	0.272	0.300	0.324	0.347	0.377	0.400
30		0.556	0.223	0.213	0.222	0.240	0.255	0.269	0.287	0.300
50		0.435	0.166	0.154	0.159	0.172	0.183	0.194	0.207	0.217
100		0.303	0.119	0.115	0.121	0.132	0.141	0.149	0.159	0.166
200		0.226	0.086	0.081	0.084	0.091	0.096	0.101	0.107	0.112
500		0.143	0.054	0.050	0.052	0.056	0.060	0.063	0.067	0.069
1000		0.096	0.038	0.036	0.037	0.040	0.042	0.044	0.047	0.049

SIMPLE POISSON PARTIAL DURATION MODEL WITH WEIBULL DISTRIBUTION AS MARGINAL (BY POME)

TABLE A.15 BIAS OF SELECTED QUANTILES : (M = 1.0 CV = 1.0 L = 2.0)

SAMPLE SIZE	T = 2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q = 1.060	2.193	2.944	3.663	4.595	5.293	5.989	6.907	7.600
10	-0.008	-0.020	-0.022	-0.023	-0.021	-0.020	-0.018	-0.015	-0.013
20	-0.004	-0.015	-0.017	-0.018	-0.019	-0.019	-0.019	-0.018	-0.017
30	-0.007	-0.013	-0.013	-0.013	-0.013	-0.012	-0.012	-0.011	-0.010
50	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
100	-0.006	-0.006	-0.006	-0.006	-0.005	-0.005	-0.005	-0.005	-0.004
200	-0.002	-0.003	-0.003	-0.003	-0.003	-0.003	-0.003	-0.003	-0.003
500	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1000	-0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

TABLE A.16 RMSE OF SELECTED QUANTILES : (M = 1.0 CV = 1.0 L = 2.0)

SAMPLE SIZE	T = 2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q = 1.060	2.193	2.944	3.663	4.595	5.293	5.989	6.907	7.600
10	0.317	0.254	0.263	0.280	0.306	0.325	0.343	0.367	0.385
20	0.228	0.185	0.193	0.206	0.223	0.236	0.249	0.264	0.274
30	0.180	0.146	0.152	0.161	0.175	0.185	0.194	0.206	0.214
50	0.142	0.118	0.123	0.131	0.142	0.149	0.157	0.166	0.172
100	0.099	0.079	0.082	0.087	0.095	0.100	0.105	0.111	0.116
200	0.070	0.057	0.060	0.064	0.069	0.073	0.076	0.081	0.084
500	0.044	0.034	0.036	0.038	0.042	0.044	0.046	0.049	0.051
1000	0.032	0.026	0.027	0.029	0.031	0.033	0.034	0.036	0.038

SIMPLE POISSON PARTIAL DURATION MODEL WITH WEIBULL DISTRIBUTION AS MARGINAL (BY POME)

TABLE A.17 BIAS OF SELECTED QUANTILES : (M = 1.0 CV = 1.0 L = 3.0)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	1.465	2.599	3.349	4.069	5.001	5.699	6.394	7.312	8.006
10		-0.008	-0.015	-0.017	-0.017	-0.016	-0.015	-0.014	-0.012	-0.011
20		-0.019	-0.021	-0.022	-0.022	-0.022	-0.022	-0.021	-0.021	-0.020
30		-0.005	-0.009	-0.010	-0.011	-0.011	-0.011	-0.011	-0.011	-0.011
50		-0.003	-0.004	-0.003	-0.003	-0.003	-0.002	-0.002	-0.002	-0.001
100		-0.001	-0.003	-0.003	-0.004	-0.004	-0.004	-0.004	-0.004	-0.004
200		0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
500		-0.001	-0.001	-0.001	-0.002	-0.002	-0.002	-0.002	-0.002	-0.002
1000		0.001	0.001	0.002	0.002	0.002	0.002	0.002	0.002	0.002

TABLE A.18 RMSE OF SELECTED QUANTILES : (M = 1.0 CV = 1.0 L = 3.0)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	1.465	2.599	3.349	4.069	5.001	5.699	6.394	7.312	8.006
10		0.220	0.212	0.227	0.244	0.268	0.285	0.302	0.322	0.337
20		0.157	0.153	0.163	0.174	0.187	0.197	0.206	0.218	0.225
30		0.130	0.123	0.129	0.137	0.147	0.154	0.161	0.169	0.175
50		0.100	0.096	0.101	0.108	0.116	0.122	0.128	0.134	0.139
100		0.071	0.067	0.071	0.075	0.081	0.085	0.089	0.094	0.097
200		0.050	0.047	0.049	0.052	0.056	0.059	0.062	0.065	0.067
500		0.032	0.030	0.032	0.034	0.036	0.038	0.040	0.042	0.043
1000		0.021	0.021	0.022	0.024	0.026	0.028	0.029	0.031	0.032

SIMPLE POISSON PARTIAL DURATION MODEL WITH WEIBULL DISTRIBUTION AS MARGINAL (BY POME)

TABLE A.19 BIAS OF SELECTED QUANTILES : (M = 1.0 CV = 1.5 L = 1.0)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	0.179	1.398	2.528	3.791	5.646	7.181	8.820	11.139	13.001
10		0.368	0.004	-0.016	-0.015	0.000	0.017	0.039	0.071	0.098
20		0.185	-0.002	-0.012	-0.012	-0.005	0.002	0.010	0.021	0.030
30		0.122	0.000	-0.007	-0.007	-0.004	0.000	0.003	0.009	0.013
50		0.071	-0.011	-0.016	-0.017	-0.017	-0.015	-0.013	-0.010	-0.008
100		0.025	-0.004	-0.006	-0.007	-0.006	-0.005	-0.004	-0.002	-0.001
200		0.032	0.002	-0.001	-0.003	-0.003	-0.003	-0.003	-0.003	-0.003
500		0.011	0.001	0.000	0.000	0.000	-0.001	-0.001	-0.001	0.000
1000		-0.003	-0.002	-0.002	-0.002	-0.001	-0.001	-0.001	-0.001	0.000

TABLE A.20 RMSE OF SELECTED QUANTILES : (M = 1.0 CV = 1.5 L = 1.0)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	0.179	1.398	2.528	3.791	5.646	7.181	8.820	11.139	13.001
10		1.417	0.555	0.538	0.599	0.763	0.944	1.174	1.549	1.885
20		0.994	0.394	0.381	0.406	0.456	0.500	0.547	0.610	0.660
30		0.834	0.327	0.313	0.328	0.358	0.382	0.406	0.436	0.459
50		0.632	0.241	0.225	0.233	0.253	0.270	0.287	0.309	0.324
100		0.441	0.174	0.168	0.177	0.193	0.206	0.218	0.233	0.244
200		0.334	0.125	0.118	0.123	0.133	0.140	0.148	0.158	0.164
500		0.209	0.078	0.074	0.076	0.082	0.087	0.092	0.098	0.102
1000		0.140	0.055	0.052	0.054	0.058	0.062	0.065	0.069	0.072

SIMPLE POISSON PARTIAL DURATION MODEL WITH WEIBULL DISTRIBUTION AS MARGINAL (BY POME)

TABLE A.21 BIAS OF SELECTED QUANTILES : (M = 1.0 CV = 1.5 L = 2.0)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	0.842	2.434	3.741	5.149	7.169	8.814	10.556	13.000	14.949
10		0.023	-0.008	-0.010	-0.007	0.000	0.006	0.013	0.022	0.029
20		0.012	-0.010	-0.013	-0.013	-0.011	-0.009	-0.006	-0.003	0.000
30		0.000	-0.011	-0.012	-0.011	-0.009	-0.007	-0.004	-0.001	0.001
50		0.008	0.008	0.009	0.012	0.014	0.017	0.019	0.021	0.023
100		-0.005	-0.007	-0.006	-0.006	-0.005	-0.004	-0.004	-0.003	-0.002
200		-0.002	-0.003	-0.003	-0.003	-0.003	-0.002	-0.002	-0.002	-0.002
500		0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.001	0.001
1000		-0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001

TABLE A.22 RMSE OF SELECTED QUANTILES : (M = 1.0 CV = 1.5 L = 2.0)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	0.842	2.434	3.741	5.149	7.169	8.814	10.556	13.000	14.949
10		0.467	0.376	0.391	0.420	0.464	0.499	0.533	0.579	0.613
20		0.335	0.270	0.282	0.302	0.330	0.351	0.371	0.396	0.415
30		0.264	0.213	0.222	0.236	0.256	0.271	0.286	0.304	0.317
50		0.209	0.173	0.181	0.193	0.210	0.222	0.233	0.247	0.257
100		0.144	0.115	0.120	0.128	0.139	0.147	0.154	0.163	0.170
200		0.102	0.083	0.087	0.093	0.101	0.106	0.112	0.118	0.123
500		0.064	0.050	0.052	0.056	0.061	0.064	0.068	0.072	0.075
1000		0.047	0.038	0.039	0.042	0.045	0.048	0.050	0.053	0.055

SIMPLE POISSON PARTIAL DURATION MODEL WITH WEIBULL DISTRIBUTION AS MARGINAL (BY POME)

TABLE A.23 BIAS OF SELECTED QUANTILES : (M = 1.0 CV = 1.5 L = 3.0)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	1.351	3.119	4.517	6.003	8.112	9.817	11.616	14.129	16.128
10		0.005	-0.007	-0.007	-0.005	0.000	0.004	0.009	0.015	0.020
20		-0.019	-0.023	-0.023	-0.022	-0.020	-0.019	-0.017	-0.014	-0.013
30		-0.002	-0.009	-0.010	-0.010	-0.009	-0.008	-0.007	-0.006	-0.005
50		-0.002	-0.002	-0.002	-0.001	0.000	0.001	0.002	0.004	0.004
100		0.000	-0.003	-0.003	-0.003	-0.003	-0.003	-0.003	-0.003	-0.003
200		0.002	0.002	0.002	0.002	0.003	0.003	0.003	0.003	0.004
500		-0.001	-0.002	-0.002	-0.002	-0.002	-0.002	-0.002	-0.002	-0.002
1000		0.001	0.002	0.002	0.003	0.003	0.003	0.003	0.004	0.004

TABLE A.24 RMSE OF SELECTED QUANTILES : (M = 1.0 CV = 1.5 L = 3.0)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	1.351	3.119	4.517	6.003	8.112	9.817	11.616	14.129	16.128
10		0.321	0.311	0.337	0.368	0.410	0.442	0.474	0.516	0.546
20		0.228	0.222	0.236	0.253	0.274	0.289	0.303	0.321	0.334
30		0.189	0.178	0.187	0.199	0.214	0.225	0.235	0.248	0.257
50		0.146	0.140	0.148	0.158	0.171	0.180	0.188	0.199	0.206
100		0.103	0.098	0.103	0.110	0.118	0.124	0.130	0.137	0.142
200		0.073	0.069	0.072	0.077	0.083	0.087	0.091	0.096	0.099
500		0.046	0.044	0.046	0.049	0.053	0.055	0.058	0.061	0.063
1000		0.031	0.030	0.033	0.035	0.038	0.040	0.042	0.045	0.046

SIMPLE POISSON PARTIAL DURATION MODEL WITH WEIBULL DISTRIBUTION AS MARGINAL (BY POME)

TABLE A.25 BIAS OF SELECTED QUANTILES : (M = 1.0 CV = 3.0 L = 1.0)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	0.028	0.869	2.331	4.576	8.883	13.254	18.665	27.528	35.605
10		1.579	0.174	0.130	0.162	0.274	0.408	0.590	0.918	1.244
20		0.814	0.082	0.059	0.069	0.102	0.134	0.170	0.222	0.265
30		0.565	0.058	0.043	0.047	0.063	0.078	0.094	0.116	0.133
50		0.335	0.014	0.001	0.001	0.008	0.015	0.023	0.035	0.044
100		0.148	0.010	0.005	0.006	0.011	0.015	0.020	0.026	0.031
200		0.114	0.011	0.005	0.004	0.004	0.005	0.006	0.008	0.010
500		0.041	0.005	0.003	0.003	0.003	0.003	0.004	0.004	0.005
1000		0.005	-0.002	-0.002	-0.001	0.000	0.000	0.001	0.002	0.002

TABLE A.26 RMSE OF SELECTED QUANTILES : (M = 1.0 CV = 3.0 L = 1.0)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	0.028	0.869	2.331	4.576	8.883	13.254	18.665	27.528	35.605
10		4.069	1.078	1.058	1.399	2.865	5.066	8.469	15.282	22.582
20		2.393	0.713	0.699	0.771	0.924	1.075	1.256	1.542	1.797
30		1.894	0.585	0.557	0.590	0.661	0.721	0.784	0.868	0.932
50		1.244	0.409	0.382	0.400	0.442	0.479	0.517	0.567	0.604
100		0.812	0.292	0.282	0.299	0.329	0.353	0.377	0.407	0.428
200		0.601	0.212	0.199	0.207	0.224	0.238	0.252	0.269	0.282
500		0.355	0.131	0.123	0.127	0.138	0.146	0.154	0.164	0.171
1000		0.235	0.091	0.087	0.090	0.097	0.103	0.108	0.115	0.120

SIMPLE POISSON PARTIAL DURATION MODEL WITH WEIBULL DISTRIBUTION AS MARGINAL (BY POME)

TABLE A.27 BIAS OF SELECTED QUANTILES : (M = 1.0 CV = 3.0 L = 2.0)

SAMPLE SIZE	T = 2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q = 0.374	2.189	4.477	7.620	13.218	18.643	25.171	35.598	44.923
10	0.156	0.064	0.067	0.083	0.114	0.140	0.169	0.209	0.240
20	0.082	0.024	0.023	0.029	0.041	0.052	0.064	0.079	0.092
30	0.039	0.007	0.008	0.013	0.022	0.030	0.037	0.048	0.056
50	0.038	0.029	0.034	0.040	0.048	0.054	0.061	0.069	0.075
100	0.002	-0.004	-0.003	-0.001	0.002	0.005	0.007	0.010	0.012
200	0.003	-0.001	-0.001	0.000	0.001	0.002	0.003	0.005	0.006
500	0.002	0.001	0.001	0.002	0.003	0.003	0.004	0.004	0.005
1000	0.000	0.000	0.001	0.001	0.001	0.002	0.002	0.002	0.003

TABLE A.28 RMSE OF SELECTED QUANTILES : (M = 1.0 CV = 3.0 L = 2.0)

SAMPLE SIZE	T = 2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q = 0.374	2.189	4.477	7.620	13.218	18.643	25.171	35.598	44.923
10	0.881	0.697	0.743	0.822	0.950	1.057	1.171	1.332	1.461
20	0.605	0.467	0.491	0.533	0.596	0.646	0.695	0.760	0.808
30	0.459	0.364	0.378	0.405	0.445	0.476	0.506	0.545	0.573
50	0.359	0.297	0.312	0.335	0.367	0.391	0.414	0.443	0.464
100	0.240	0.192	0.201	0.214	0.234	0.248	0.262	0.278	0.290
200	0.170	0.139	0.146	0.155	0.169	0.178	0.188	0.199	0.207
500	0.106	0.084	0.088	0.094	0.102	0.108	0.114	0.121	0.125
1000	0.077	0.063	0.066	0.070	0.076	0.080	0.084	0.089	0.092

SIMPLE POISSON PARTIAL DURATION MODEL WITH WEIBULL DISTRIBUTION AS MARGINAL (BY POME)

TABLE A.29 BIAS OF SELECTED QUANTILES : (M = 1.0 CV = 3.0 L = 3.0)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	0.821	3.306	6.126	9.835	16.235	22.308	29.516	40.894	50.973
10		0.065	0.041	0.050	0.064	0.089	0.109	0.131	0.160	0.183
20		-0.003	-0.012	-0.008	-0.002	0.008	0.015	0.022	0.032	0.040
30		0.016	0.003	0.003	0.006	0.010	0.014	0.018	0.024	0.028
50		0.009	0.007	0.010	0.012	0.017	0.020	0.023	0.027	0.031
100		0.006	0.001	0.001	0.001	0.002	0.003	0.004	0.005	0.006
200		0.006	0.006	0.006	0.007	0.008	0.009	0.010	0.011	0.011
500		-0.001	-0.002	-0.002	-0.002	-0.002	-0.002	-0.002	-0.001	-0.001
1000		0.003	0.004	0.005	0.005	0.006	0.006	0.007	0.007	0.007

TABLE A.30 RMSE OF SELECTED QUANTILES : (M = 1.0 CV = 3.0 L = 3.0)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	0.821	3.306	6.126	9.835	16.235	22.308	29.516	40.894	50.973
10		0.560	0.551	0.621	0.710	0.843	0.954	1.072	1.238	1.372
20		0.386	0.373	0.400	0.432	0.476	0.508	0.539	0.580	0.609
30		0.318	0.298	0.315	0.335	0.363	0.384	0.404	0.429	0.448
50		0.246	0.236	0.252	0.269	0.293	0.310	0.326	0.346	0.360
100		0.173	0.163	0.173	0.184	0.198	0.208	0.218	0.230	0.239
200		0.123	0.115	0.121	0.129	0.139	0.146	0.153	0.162	0.167
500		0.077	0.073	0.077	0.082	0.088	0.092	0.096	0.101	0.105
1000		0.052	0.051	0.055	0.059	0.064	0.068	0.071	0.075	0.078

SIMPLE POISSON PARTIAL DURATION MODEL WITH WEIBULL DISTRIBUTION AS MARGINAL (BY PWM)

TABLE A.31 BIAS OF SELECTED QUANTILES : (M = 1.0 CV = 0.5 L = 1.0)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	0.700	1.369	1.661	1.895	2.158	2.334	2.496	2.693	2.832
10		-0.341	0.018	0.180	0.311	0.458	0.559	0.651	0.766	0.848
20		-0.220	0.027	0.103	0.158	0.215	0.251	0.283	0.320	0.345
30		-0.159	0.023	0.072	0.106	0.141	0.162	0.181	0.203	0.218
50		-0.098	0.013	0.041	0.060	0.079	0.091	0.101	0.113	0.121
100		-0.049	0.007	0.021	0.030	0.038	0.044	0.049	0.054	0.057
200		-0.019	0.005	0.011	0.015	0.019	0.022	0.024	0.026	0.028
500		-0.008	0.002	0.005	0.006	0.008	0.009	0.010	0.011	0.012
1000		-0.007	0.000	0.002	0.003	0.004	0.004	0.005	0.005	0.006

TABLE A.32 RMSE OF SELECTED QUANTILES : (M = 1.0 CV = 0.5 L = 1.0)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	0.700	1.369	1.661	1.895	2.158	2.334	2.496	2.693	2.832
10		0.587	0.274	0.296	0.390	0.540	0.658	0.777	0.938	1.062
20		0.452	0.159	0.171	0.213	0.270	0.311	0.348	0.394	0.426
30		0.367	0.120	0.130	0.156	0.190	0.214	0.235	0.261	0.279
50		0.266	0.085	0.086	0.099	0.117	0.130	0.142	0.157	0.167
100		0.166	0.059	0.060	0.066	0.075	0.082	0.088	0.095	0.100
200		0.112	0.041	0.041	0.043	0.048	0.052	0.055	0.059	0.062
500		0.070	0.026	0.025	0.026	0.029	0.031	0.032	0.035	0.036
1000		0.047	0.018	0.017	0.018	0.020	0.021	0.022	0.024	0.025

SIMPLE POISSON PARTIAL DURATION MODEL WITH WEIBULL DISTRIBUTION AS MARGINAL (BY PWM)

TABLE A.33 BIAS OF SELECTED QUANTILES : (M = 1.0 CV = 0.5 L = 2.0)

SAMPLE SIZE	T = 2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q = 1.161	1.641	1.887	2.094	2.333	2.495	2.646	2.832	2.964
10	-0.033	0.101	0.159	0.204	0.253	0.285	0.313	0.347	0.370
20	-0.010	0.050	0.075	0.093	0.113	0.126	0.137	0.150	0.159
30	-0.008	0.033	0.050	0.062	0.076	0.084	0.091	0.100	0.106
50	-0.002	0.024	0.035	0.043	0.051	0.056	0.061	0.066	0.070
100	-0.004	0.009	0.014	0.017	0.021	0.024	0.026	0.028	0.030
200	-0.001	0.005	0.007	0.009	0.011	0.012	0.013	0.014	0.015
500	0.000	0.002	0.003	0.004	0.005	0.005	0.006	0.006	0.007
1000	-0.001	0.001	0.002	0.002	0.002	0.003	0.003	0.003	0.003

TABLE A.34 RMSE OF SELECTED QUANTILES : (M = 1.0 CV = 0.5 L = 2.0)

SAMPLE SIZE	T = 2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q = 1.161	1.641	1.887	2.094	2.333	2.495	2.646	2.832	2.964
10	0.196	0.168	0.211	0.256	0.309	0.346	0.380	0.421	0.450
20	0.122	0.105	0.122	0.141	0.163	0.178	0.192	0.209	0.220
30	0.093	0.079	0.090	0.102	0.116	0.127	0.136	0.147	0.155
50	0.071	0.062	0.069	0.077	0.086	0.093	0.099	0.106	0.111
100	0.048	0.039	0.042	0.046	0.051	0.055	0.058	0.062	0.065
200	0.034	0.028	0.030	0.032	0.035	0.037	0.039	0.042	0.044
500	0.021	0.017	0.017	0.019	0.021	0.022	0.023	0.024	0.025
1000	0.015	0.012	0.013	0.014	0.015	0.016	0.017	0.018	0.018

SIMPLE POISSON PARTIAL DURATION MODEL WITH WEIBULL DISTRIBUTION AS MARGINAL (BY PWM)

TABLE A.35 BIAS OF SELECTED QUANTILES : (M = 1.0 CV = 0.5 L = 3.0)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	1.354	1.779	2.007	2.202	2.429	2.585	2.730	2.910	3.038
10		0.023	0.090	0.120	0.145	0.171	0.188	0.204	0.222	0.234
20		0.006	0.038	0.052	0.064	0.075	0.083	0.090	0.098	0.103
30		0.008	0.028	0.037	0.044	0.051	0.056	0.060	0.065	0.069
50		0.005	0.018	0.023	0.028	0.032	0.035	0.038	0.041	0.043
100		0.003	0.009	0.011	0.013	0.015	0.017	0.018	0.019	0.020
200		0.002	0.005	0.007	0.008	0.009	0.009	0.010	0.011	0.011
500		0.000	0.001	0.002	0.002	0.002	0.003	0.003	0.003	0.003
1000		0.001	0.002	0.002	0.002	0.003	0.003	0.003	0.003	0.004

TABLE A.36 RMSE OF SELECTED QUANTILES : (M = 1.0 CV = 0.5 L = 3.0)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	1.354	1.779	2.007	2.202	2.429	2.585	2.730	2.910	3.038
10		0.120	0.139	0.166	0.192	0.222	0.242	0.261	0.283	0.299
20		0.078	0.085	0.097	0.109	0.123	0.132	0.141	0.151	0.158
30		0.065	0.066	0.074	0.081	0.090	0.096	0.102	0.109	0.114
50		0.049	0.050	0.054	0.059	0.065	0.069	0.073	0.078	0.081
100		0.034	0.034	0.036	0.039	0.043	0.045	0.047	0.050	0.052
200		0.024	0.023	0.025	0.027	0.029	0.030	0.032	0.034	0.035
500		0.015	0.014	0.015	0.016	0.017	0.018	0.019	0.020	0.021
1000		0.010	0.010	0.011	0.012	0.013	0.013	0.014	0.015	0.015

SIMPLE POISSON PARTIAL DURATION MODEL WITH WEIBULL DISTRIBUTION AS MARGINAL (BY PWM)

TABLE A.37 BIAS OF SELECTED QUANTILES : (M = 1.0 CV = 1.0 L = 1.0)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	0.367	1.500	2.250	2.970	3.902	4.600	5.296	6.214	6.907
10		-0.268	-0.093	0.042	0.163	0.309	0.414	0.516	0.647	0.745
20		-0.180	-0.039	0.032	0.087	0.148	0.188	0.225	0.269	0.300
30		-0.143	-0.023	0.027	0.065	0.106	0.132	0.155	0.183	0.202
50		-0.100	-0.019	0.011	0.033	0.056	0.071	0.084	0.099	0.110
100		-0.055	-0.008	0.005	0.015	0.026	0.032	0.038	0.044	0.049
200		-0.018	-0.001	0.005	0.010	0.015	0.018	0.021	0.024	0.026
500		-0.009	0.000	0.002	0.004	0.006	0.008	0.009	0.010	0.011
1000		-0.010	-0.002	0.000	0.001	0.003	0.003	0.004	0.005	0.005

TABLE A.38 RMSE OF SELECTED QUANTILES : (M = 1.0 CV = 1.0 L = 1.0)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	0.367	1.500	2.250	2.970	3.902	4.600	5.296	6.214	6.907
10		0.779	0.429	0.408	0.466	0.605	0.736	0.881	1.092	1.264
20		0.622	0.289	0.271	0.298	0.356	0.405	0.456	0.522	0.571
30		0.546	0.232	0.223	0.246	0.288	0.321	0.353	0.394	0.424
50		0.433	0.171	0.159	0.169	0.192	0.212	0.230	0.254	0.271
100		0.307	0.121	0.116	0.123	0.137	0.148	0.159	0.172	0.181
200		0.226	0.086	0.082	0.086	0.095	0.101	0.108	0.115	0.121
500		0.145	0.054	0.051	0.053	0.058	0.063	0.066	0.071	0.074
1000		0.098	0.038	0.036	0.038	0.042	0.044	0.047	0.050	0.052

SIMPLE POISSON PARTIAL DURATION MODEL WITH WEIBULL DISTRIBUTION AS MARGINAL (BY PWM)

TABLE A.39 BIAS OF SELECTED QUANTILES : (M = 1.0 CV = 1.0 L = 2.0)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	1.060	2.193	2.944	3.663	4.595	5.293	5.989	6.907	7.600
10		-0.088	0.032	0.091	0.141	0.197	0.234	0.269	0.312	0.343
20		-0.041	0.014	0.040	0.060	0.083	0.098	0.112	0.128	0.139
30		-0.033	0.008	0.026	0.041	0.057	0.068	0.078	0.089	0.097
50		-0.014	0.014	0.027	0.037	0.047	0.054	0.060	0.067	0.072
100		-0.014	0.001	0.008	0.013	0.018	0.021	0.024	0.028	0.030
200		-0.006	0.001	0.003	0.005	0.008	0.009	0.010	0.012	0.013
500		-0.002	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.007
1000		-0.002	0.000	0.001	0.002	0.002	0.002	0.003	0.003	0.003

TABLE A.40 RMSE OF SELECTED QUANTILES : (M = 1.0 CV = 1.0 L = 2.0)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	1.060	2.193	2.944	3.663	4.595	5.293	5.989	6.907	7.600
10		0.347	0.272	0.301	0.348	0.418	0.472	0.526	0.595	0.646
20		0.240	0.189	0.203	0.224	0.255	0.277	0.298	0.325	0.344
30		0.188	0.149	0.158	0.174	0.196	0.213	0.229	0.249	0.263
50		0.145	0.120	0.129	0.141	0.157	0.169	0.179	0.193	0.202
100		0.100	0.080	0.084	0.091	0.101	0.108	0.114	0.122	0.127
200		0.070	0.057	0.060	0.064	0.070	0.075	0.079	0.084	0.087
500		0.044	0.035	0.036	0.039	0.043	0.045	0.048	0.051	0.053
1000		0.032	0.026	0.027	0.029	0.031	0.033	0.035	0.037	0.038

SIMPLE POISSON PARTIAL DURATION MODEL WITH WEIBULL DISTRIBUTION AS MARGINAL (BY PWM)

TABLE A.41 BIAS OF SELECTED QUANTILES : (M = 1.0 CV = 1.0 L = 3.0)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	1.465	2.599	3.349	4.069	5.001	5.699	6.394	7.312	8.006
10		-0.021	0.045	0.078	0.105	0.136	0.157	0.176	0.199	0.215
20		-0.024	0.011	0.027	0.041	0.055	0.065	0.074	0.084	0.091
30		-0.009	0.011	0.021	0.029	0.038	0.044	0.049	0.056	0.060
50		-0.005	0.008	0.015	0.020	0.025	0.029	0.032	0.036	0.038
100		-0.002	0.004	0.007	0.010	0.012	0.014	0.016	0.018	0.019
200		0.000	0.004	0.006	0.007	0.009	0.010	0.011	0.012	0.012
500		-0.001	0.000	0.001	0.001	0.001	0.002	0.002	0.002	0.003
1000		0.001	0.002	0.002	0.003	0.003	0.003	0.004	0.004	0.004

TABLE A.42 RMSE OF SELECTED QUANTILES : (M = 1.0 CV = 1.0 L = 3.0)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	1.465	2.599	3.349	4.069	5.001	5.699	6.394	7.312	8.006
10		0.231	0.221	0.248	0.280	0.322	0.353	0.383	0.421	0.449
20		0.160	0.157	0.172	0.189	0.211	0.226	0.241	0.260	0.273
30		0.132	0.125	0.135	0.146	0.161	0.171	0.181	0.194	0.203
50		0.101	0.098	0.105	0.113	0.124	0.131	0.139	0.147	0.154
100		0.071	0.068	0.073	0.079	0.086	0.091	0.095	0.101	0.105
200		0.050	0.048	0.051	0.055	0.060	0.063	0.066	0.070	0.072
500		0.032	0.030	0.032	0.034	0.037	0.039	0.041	0.043	0.045
1000		0.021	0.021	0.022	0.024	0.026	0.028	0.029	0.031	0.032

SIMPLE POISSON PARTIAL DURATION MODEL WITH WEIBULL DISTRIBUTION AS MARGINAL (BY PWM)

TABLE A.43 BIAS OF SELECTED QUANTILES : (M = 1.0 CV = 1.5 L = 1.0)

SAMPLE SIZE	T = 2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q = 0.179	1.398	2.528	3.791	5.646	7.181	8.820	11.139	13.001
10	-0.115	-0.154	-0.048	0.061	0.206	0.318	0.432	0.588	0.709
20	-0.083	-0.079	-0.018	0.037	0.102	0.149	0.193	0.248	0.289
30	-0.074	-0.053	-0.005	0.036	0.084	0.116	0.146	0.184	0.210
50	-0.062	-0.041	-0.012	0.013	0.041	0.060	0.078	0.099	0.114
100	-0.037	-0.020	-0.006	0.005	0.017	0.025	0.032	0.041	0.047
200	-0.007	-0.006	0.001	0.007	0.013	0.017	0.021	0.026	0.029
500	-0.003	-0.002	0.001	0.003	0.005	0.006	0.008	0.009	0.010
1000	-0.011	-0.004	-0.001	0.000	0.002	0.003	0.004	0.005	0.006

TABLE A.44 RMSE OF SELECTED QUANTILES : (M = 1.0 CV = 1.5 L = 1.0)

SAMPLE SIZE	T = 2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q = 0.179	1.398	2.528	3.791	5.646	7.181	8.820	11.139	13.001
10	1.117	0.557	0.531	0.584	0.735	0.892	1.078	1.370	1.625
20	0.856	0.398	0.375	0.403	0.475	0.542	0.614	0.714	0.792
30	0.755	0.330	0.316	0.345	0.406	0.457	0.509	0.579	0.630
50	0.596	0.244	0.228	0.245	0.281	0.312	0.343	0.384	0.413
100	0.440	0.174	0.167	0.179	0.202	0.220	0.237	0.259	0.274
200	0.332	0.125	0.120	0.128	0.142	0.153	0.164	0.177	0.186
500	0.214	0.079	0.074	0.079	0.088	0.095	0.101	0.109	0.114
1000	0.146	0.055	0.053	0.057	0.063	0.068	0.072	0.078	0.082

SIMPLE POISSON PARTIAL DURATION MODEL WITH WEIBULL DISTRIBUTION AS MARGINAL (BY PWM)

TABLE A.45 BIAS OF SELECTED QUANTILES : (M = 1.0 CV = 1.5 L = 2.0)

SAMPLE SIZE	T = 2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q = 0.842	2.434	3.741	5.149	7.169	8.814	10.556	13.000	14.949
10	-0.110	-0.017	0.042	0.096	0.162	0.209	0.254	0.312	0.355
20	-0.055	-0.012	0.015	0.038	0.066	0.084	0.102	0.124	0.140
30	-0.046	-0.012	0.009	0.027	0.047	0.061	0.075	0.091	0.102
50	-0.020	0.007	0.022	0.034	0.047	0.057	0.065	0.075	0.082
100	-0.022	-0.004	0.004	0.011	0.018	0.023	0.027	0.032	0.036
200	-0.009	-0.003	0.000	0.003	0.006	0.007	0.009	0.011	0.013
500	-0.003	0.000	0.001	0.003	0.004	0.005	0.006	0.007	0.008
1000	-0.002	0.000	0.001	0.001	0.002	0.002	0.003	0.003	0.004

TABLE A.46 RMSE OF SELECTED QUANTILES : (M = 1.0 CV = 1.5 L = 2.0)

SAMPLE SIZE	T = 2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q = 0.842	2.434	3.741	5.149	7.169	8.814	10.556	13.000	14.949
10	0.468	0.381	0.415	0.477	0.581	0.667	0.758	0.884	0.982
20	0.340	0.271	0.289	0.320	0.366	0.401	0.436	0.481	0.514
30	0.270	0.215	0.230	0.256	0.294	0.323	0.351	0.388	0.414
50	0.210	0.175	0.190	0.209	0.235	0.255	0.273	0.296	0.312
100	0.145	0.117	0.125	0.137	0.153	0.164	0.175	0.188	0.197
200	0.103	0.084	0.089	0.097	0.107	0.114	0.121	0.129	0.135
500	0.064	0.051	0.054	0.059	0.065	0.070	0.074	0.079	0.082
1000	0.047	0.038	0.040	0.043	0.047	0.050	0.053	0.056	0.059

SIMPLE POISSON PARTIAL DURATION MODEL WITH WEIBULL DISTRIBUTION AS MARGINAL (BY PWM)

TABLE A.47 BIAS OF SELECTED QUANTILES : (M = 1.0 CV = 1.5 L = 3.0)

SAMPLE SIZE	T = 2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q = 1.351	3.119	4.517	6.003	8.112	9.817	11.616	14.129	16.128
10	-0.048	0.012	0.048	0.079	0.117	0.143	0.168	0.199	0.222
20	-0.046	-0.009	0.010	0.027	0.046	0.059	0.071	0.086	0.097
30	-0.020	-0.001	0.009	0.019	0.030	0.038	0.045	0.053	0.060
50	-0.012	0.001	0.008	0.014	0.021	0.026	0.030	0.036	0.039
100	-0.005	0.001	0.005	0.008	0.012	0.014	0.017	0.019	0.021
200	-0.001	0.003	0.006	0.007	0.009	0.011	0.012	0.013	0.014
500	-0.002	-0.001	0.000	0.000	0.001	0.002	0.002	0.002	0.003
1000	0.001	0.002	0.002	0.002	0.003	0.003	0.003	0.004	0.004

TABLE A.48 RMSE OF SELECTED QUANTILES : (M = 1.0 CV = 1.5 L = 3.0)

SAMPLE SIZE	T = 2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q = 1.351	3.119	4.517	6.003	8.112	9.817	11.616	14.129	16.128
10	0.325	0.309	0.346	0.393	0.463	0.517	0.572	0.645	0.700
20	0.231	0.227	0.250	0.277	0.314	0.342	0.369	0.403	0.429
30	0.190	0.181	0.196	0.215	0.239	0.256	0.273	0.295	0.310
50	0.147	0.143	0.155	0.170	0.188	0.201	0.213	0.228	0.239
100	0.103	0.101	0.109	0.119	0.131	0.140	0.148	0.158	0.165
200	0.073	0.071	0.077	0.083	0.091	0.097	0.102	0.108	0.113
500	0.046	0.045	0.049	0.053	0.058	0.061	0.064	0.068	0.070
1000	0.031	0.031	0.034	0.036	0.040	0.042	0.044	0.047	0.049

SIMPLE POISSON PARTIAL DURATION MODEL WITH WEIBULL DISTRIBUTION AS MARGINAL (BY PWM)

TABLE A.49 BIAS OF SELECTED QUANTILES : (M = 1.0 CV = 3.0 L = 1.0)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	0.028	0.869	2.331	4.576	8.883	13.254	18.665	27.528	35.605
10		0.753	-0.193	-0.182	-0.124	-0.012	0.096	0.222	0.420	0.597
20		0.436	-0.121	-0.110	-0.072	-0.005	0.054	0.119	0.213	0.291
30		0.326	-0.086	-0.069	-0.032	0.027	0.077	0.129	0.203	0.261
50		0.204	-0.077	-0.066	-0.041	-0.002	0.030	0.062	0.107	0.141
100		0.109	-0.038	-0.034	-0.022	-0.005	0.010	0.024	0.043	0.057
200		0.083	-0.015	-0.012	-0.004	0.007	0.016	0.024	0.035	0.044
500		0.040	-0.004	-0.005	-0.003	0.000	0.002	0.004	0.008	0.010
1000		0.002	-0.007	-0.005	-0.003	0.000	0.003	0.005	0.007	0.009

TABLE A.50 RMSE OF SELECTED QUANTILES : (M = 1.0 CV = 3.0 L = 1.0)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	0.028	0.869	2.331	4.576	8.883	13.254	18.665	27.528	35.605
10		3.337	0.861	0.792	0.826	0.978	1.174	1.447	1.943	2.439
20		2.084	0.632	0.584	0.615	0.725	0.847	0.999	1.244	1.466
30		1.705	0.538	0.508	0.553	0.672	0.793	0.938	1.164	1.362
50		1.219	0.397	0.367	0.397	0.476	0.551	0.634	0.752	0.845
100		0.853	0.288	0.272	0.298	0.352	0.398	0.445	0.507	0.555
200		0.637	0.212	0.198	0.216	0.252	0.281	0.309	0.346	0.372
500		0.393	0.132	0.124	0.136	0.158	0.175	0.191	0.210	0.224
1000		0.270	0.093	0.089	0.099	0.117	0.129	0.141	0.156	0.166

SIMPLE POISSON PARTIAL DURATION MODEL WITH WEIBULL DISTRIBUTION AS MARGINAL (BY PWM)

TABLE A.51 BIAS OF SELECTED QUANTILES : (M = 1.0 CV = 3.0 L = 2.0)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	0.374	2 189	4.477	7.620	13.219	18.643	25.171	35.598	44.923
10		-0.071	-0.104	-0.065	-0.012	0.074	0.147	0.227	0.345	0.442
20		-0.038	-0.065	-0.045	-0.017	0.024	0.058	0.093	0.140	0.177
30		-0.043	-0.055	-0.035	-0.010	0.026	0.056	0.086	0.126	0.157
50		-0.012	-0.011	0.005	0.023	0.047	0.066	0.084	0.108	0.126
100		-0.032	-0.020	-0.008	0.004	0.018	0.028	0.038	0.051	0.060
200		-0.010	-0.010	-0.007	-0.002	0.003	0.008	0.012	0.017	0.021
500		-0.003	-0.003	-0.002	0.000	0.002	0.004	0.006	0.008	0.009
1000		-0.003	-0.002	-0.001	0.000	0.001	0.002	0.003	0.004	0.004

TABLE A.52 RMSE OF SELECTED QUANTILES : (M = 1.0 CV = 3.0 L = 2.0)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	0.374	2.189	4.477	7.620	13.219	18.643	25.171	35.598	44.923
10		0.776	0.601	0.649	0.768	1.020	1.283	1.609	2.144	2.633
20		0.572	0.440	0.470	0.532	0.640	0.736	0.842	0.995	1.120
30		0.457	0.349	0.382	0.444	0.548	0.637	0.731	0.864	0.971
50		0.360	0.288	0.320	0.368	0.438	0.494	0.550	0.626	0.683
100		0.248	0.193	0.213	0.243	0.284	0.314	0.344	0.381	0.408
200		0.181	0.140	0.155	0.176	0.204	0.224	0.243	0.266	0.283
500		0.112	0.085	0.095	0.108	0.124	0.136	0.147	0.160	0.168
1000		0.084	0.063	0.069	0.078	0.089	0.097	0.104	0.113	0.119

SIMPLE POISSON PARTIAL DURATION MODEL WITH WEIBULL DISTRIBUTION AS MARGINAL (BY PWM)

TABLE A.53 BIAS OF SELECTED QUANTILES : (M = 1.0 CV = 3.0 L = 3.0)

SAMPLE SIZE	T = 2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q = 0.821	3.306	6.126	9.835	16.235	22.308	29.516	40.894	50.973
10	-0.078	-0.063	-0.028	0.013	0.073	0.122	0.173	0.246	0.303
20	-0.080	-0.057	-0.032	-0.005	0.032	0.060	0.088	0.125	0.154
30	-0.035	-0.033	-0.021	-0.006	0.014	0.029	0.045	0.066	0.081
50	-0.022	-0.018	-0.009	0.001	0.014	0.025	0.035	0.048	0.058
100	-0.011	-0.007	-0.002	0.005	0.013	0.019	0.025	0.033	0.038
200	-0.002	0.000	0.003	0.006	0.010	0.012	0.015	0.018	0.021
500	-0.005	-0.003	-0.002	0.000	0.001	0.003	0.004	0.005	0.006
1000	0.002	0.001	0.001	0.001	0.001	0.002	0.002	0.002	0.002

TABLE A.54 RMSE OF SELECTED QUANTILES : (M = 1.0 CV = 3.0 L = 3.0)

SAMPLE SIZE	T = 2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q = 0.821	3.306	6.126	9.835	16.235	22.308	29.516	40.894	50.973
10	0.519	0.479	0.545	0.656	0.867	1.078	1.336	1.756	2.136
20	0.380	0.368	0.418	0.485	0.587	0.671	0.759	0.882	0.979
30	0.315	0.299	0.335	0.380	0.444	0.494	0.544	0.611	0.661
50	0.246	0.238	0.270	0.306	0.356	0.394	0.430	0.477	0.511
100	0.176	0.169	0.192	0.218	0.252	0.276	0.299	0.328	0.348
200	0.124	0.120	0.135	0.152	0.173	0.187	0.201	0.218	0.230
500	0.078	0.078	0.088	0.099	0.112	0.121	0.129	0.139	0.146
1000	0.053	0.053	0.060	0.067	0.076	0.082	0.087	0.094	0.098

COMPOUND (SEPARATED) POISSON PARTIAL DURATION MODEL WITH EXPONENTIAL DISTRIBUTIONS AS MARGINALS

TABLE A.55 BIAS OF SELECTED QUANTILES : (M1 = 1.0 M2 = 1.5 L1 = 0.7 L2 = 0.3)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	0.409	1.707	2.596	3.474	4.650	5.562	6.494	7.757	8.734
10		-0.073	-0.017	0.026	0.067	0.108	0.130	0.144	0.155	0.159
20		-0.081	-0.027	0.001	0.028	0.056	0.071	0.080	0.087	0.090
30		-0.079	-0.031	-0.012	0.004	0.024	0.035	0.042	0.047	0.049
50		-0.035	-0.010	-0.001	0.009	0.019	0.026	0.029	0.032	0.032
100		-0.025	-0.012	-0.007	-0.002	0.003	0.007	0.010	0.011	0.011
200		0.003	0.002	0.003	0.006	0.009	0.011	0.013	0.013	0.013
500		-0.005	0.000	0.001	0.002	0.003	0.004	0.004	0.004	0.004
1000		-0.005	-0.001	0.000	0.001	0.002	0.002	0.003	0.003	0.003

TABLE A.56 RMSE OF SELECTED QUANTILES : (M1 = 1.0 M2 = 1.5 L1 = 0.7 L2 = 0.3)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	0.409	1.707	2.596	3.474	4.650	5.562	6.494	7.757	8.734
10		0.925	0.393	0.377	0.396	0.437	0.469	0.494	0.517	0.530
20		0.660	0.285	0.272	0.284	0.315	0.337	0.354	0.370	0.379
30		0.542	0.229	0.211	0.217	0.236	0.251	0.265	0.281	0.286
50		0.421	0.170	0.158	0.161	0.174	0.186	0.197	0.207	0.214
100		0.292	0.122	0.112	0.112	0.119	0.126	0.133	0.141	0.146
200		0.207	0.088	0.083	0.085	0.092	0.098	0.105	0.112	0.116
500		0.135	0.057	0.053	0.053	0.057	0.060	0.064	0.069	0.072
1000		0.094	0.038	0.035	0.035	0.038	0.040	0.043	0.046	0.048

COMPOUND (SEPARATED) POISSON PARTIAL DURATION MODEL WITH EXPONENTIAL DISTRIBUTIONS AS MARGINALS

TABLE A.57 BIAS OF SELECTED QUANTILES : (M1 = 1.0 M2 = 1.5 L1 = 1.3 L2 = 0.7)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	1.222	2.584	3.521	4.448	5.687	6.642	6.579	8.921	9.924
10		-0.034	0.005	0.030	0.051	0.069	0.077	0.252	0.085	0.085
20		-0.031	-0.009	0.005	0.017	0.027	0.031	0.197	0.035	0.035
30		-0.011	0.000	0.007	0.014	0.020	0.022	0.184	0.024	0.023
50		-0.007	-0.001	0.003	0.007	0.010	0.011	0.170	0.011	0.010
100		-0.002	0.003	0.005	0.007	0.009	0.010	0.169	0.010	0.010
200		0.002	0.002	0.002	0.004	0.004	0.005	0.163	0.004	0.004
500		-0.001	0.000	0.001	0.000	0.003	0.003	0.161	0.003	0.003
1000		-0.001	-0.001	-0.001	0.000	-0.001	0.000	0.157	0.000	0.000

TABLE A.58 RMSE OF SELECTED QUANTILES : (M1 = 1.0 M2 = 1.5 L1 = 1.3 L2 = 0.7)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	1.222	2.584	3.521	4.448	5.687	6.642	6.579	8.921	9.924
10		0.318	0.266	0.278	0.298	0.322	0.337	0.465	0.358	0.364
20		0.223	0.182	0.184	0.195	0.210	0.219	0.326	0.235	0.239
30		0.182	0.150	0.152	0.159	0.170	0.177	0.279	0.189	0.192
50		0.143	0.116	0.117	0.122	0.130	0.136	0.235	0.146	0.149
100		0.102	0.083	0.084	0.087	0.094	0.098	0.206	0.107	0.110
200		0.073	0.060	0.063	0.063	0.068	0.071	0.184	0.077	0.080
500		0.045	0.036	0.036	0.056	0.039	0.041	0.168	0.046	0.047
1000		0.031	0.025	0.025	0.026	0.034	0.029	0.161	0.033	0.033

COMPOUND (SEPARATED) POISSON PARTIAL DURATION MODEL WITH EXPONENTIAL DISTRIBUTIONS AS MARGINALS

TABLE A.59 BIAS OF SELECTED QUANTILES : (M1 = 1.0 M2 = 1.5 L1 = 2.0 L2 = 1.0)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	1.690	3.064	4.009	4.944	6.192	7.154	8.131	9.444	10.450
10		-0.008	0.020	0.037	0.051	0.061	0.066	0.068	0.070	0.070
20		-0.012	0.001	0.009	0.016	0.020	0.022	0.023	0.022	0.021
30		-0.012	-0.002	0.003	0.008	0.012	0.014	0.014	0.014	0.013
50		-0.005	-0.001	0.002	0.005	0.008	0.009	0.009	0.009	0.008
100		-0.001	0.001	0.003	0.004	0.005	0.005	0.005	0.005	0.004
200		0.001	0.002	0.003	0.003	0.004	0.004	0.004	0.004	0.004
500		-0.001	0.000	0.000	0.001	0.001	0.001	0.001	0.002	0.001
1000		-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001

TABLE A.60 RMSE OF SELECTED QUANTILES : (M1 = 1.0 M2 = 1.5 L1 = 2.0 L2 = 1.0)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	1.690	3.064	4.009	4.944	6.192	7.154	8.131	9.444	10.450
10		0.229	0.219	0.234	0.251	0.268	0.279	0.286	0.293	0.297
20		0.159	0.149	0.156	0.165	0.175	0.182	0.187	0.193	0.196
30		0.130	0.117	0.121	0.127	0.137	0.144	0.149	0.154	0.158
50		0.102	0.094	0.097	0.102	0.109	0.115	0.119	0.124	0.126
100		0.072	0.068	0.071	0.075	0.081	0.085	0.089	0.093	0.095
200		0.052	0.045	0.047	0.049	0.053	0.057	0.059	0.062	0.064
500		0.031	0.028	0.029	0.030	0.032	0.034	0.035	0.037	0.038
1000		0.023	0.022	0.023	0.024	0.025	0.027	0.028	0.029	0.030

COMPOUND (SEPARATED) POISSON PARTIAL DURATION MODEL WITH EXPONENTIAL DISTRIBUTIONS AS MARGINALS

TABLE A.61 BIAS OF SELECTED QUANTILES : (M1 = 1.0 M2 = 2.0 L1 = 0.7 L2 = 0.3)

SAMPLE SIZE	T = 2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q = 0.437	1.885	2.949	4.066	5.655	6.933	8.257	10.050	11.422
10	-0.063	-0.006	0.040	0.077	0.103	0.109	0.109	0.107	0.104
20	-0.076	-0.025	0.007	0.034	0.050	0.053	0.051	0.048	0.045
30	-0.075	-0.030	-0.012	0.006	0.018	0.019	0.018	0.015	0.013
50	-0.031	-0.010	0.000	0.009	0.014	0.014	0.011	0.008	0.006
100	-0.024	-0.013	-0.007	-0.002	0.001	0.002	0.000	-0.002	-0.003
200	0.004	0.002	0.004	0.007	0.009	0.008	0.007	0.006	0.005
500	-0.005	-0.001	-0.001	0.001	0.002	0.001	0.001	-0.001	0.000
1000	-0.005	-0.001	-0.001	0.001	0.001	0.001	0.002	0.001	0.001

TABLE A.62 RMSE OF SELECTED QUANTILES : (M1 = 1.0 M2 = 2.0 L1 = 0.7 L2 = 0.3)

SAMPLE SIZE	T = 2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q = 0.437	1.885	2.949	4.066	5.655	6.933	8.257	10.050	11.422
10	0.942	0.421	0.425	0.457	0.499	0.522	0.536	0.550	0.557
20	0.666	0.302	0.305	0.335	0.370	0.388	0.399	0.408	0.412
30	0.547	0.239	0.233	0.254	0.282	0.297	0.308	0.316	0.321
50	0.426	0.179	0.175	0.190	0.213	0.226	0.234	0.242	0.245
100	0.294	0.127	0.122	0.130	0.145	0.155	0.161	0.167	0.169
200	0.209	0.093	0.091	0.100	0.114	0.123	0.128	0.132	0.134
500	0.136	0.059	0.059	0.062	0.071	0.077	0.081	0.084	0.085
1000	0.095	0.040	0.043	0.042	0.049	0.052	0.055	0.057	0.057

COMPOUND (SEPARATED) POISSON PARTIAL DURATION MODEL WITH EXPONENTIAL DISTRIBUTIONS AS MARGINALS

TABLE A.63 BIAS OF SELECTED QUANTILES : (M1 = 1.0 M2 = 2.0 L1 = 1.3 L2 = 0.7)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	1.351	2.984	4.198	5.456	6.939	8.538	9.904	11.724	13.106
10		-0.031	0.011	0.034	0.047	0.087	0.049	0.048	0.046	0.044
20		-0.030	-0.007	0.006	0.014	0.051	0.014	0.012	0.010	0.009
30		-0.012	0.000	0.008	0.011	0.047	0.009	0.007	0.006	0.005
50		-0.006	0.000	0.004	0.005	0.039	0.002	0.000	-0.002	-0.003
100		-0.001	0.002	0.004	0.006	0.042	0.003	0.002	0.003	0.002
200		0.002	0.002	0.002	0.003	0.038	0.001	0.001	0.000	0.001
500		-0.001	0.000	0.001	0.002	0.037	0.001	0.001	0.002	0.001
1000		-0.002	-0.002	-0.001	-0.001	0.034	-0.002	-0.002	-0.001	-0.002

TABLE A.64 RMSE OF SELECTED QUANTILES : (M1 = 1.0 M2 = 2.0 L1 = 1.3 L2 = 0.7)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	1.351	2.984	4.198	5.456	6.939	8.538	9.904	11.724	13.106
10		0.332	0.299	0.325	0.345	0.384	0.373	0.380	0.386	0.389
20		0.231	0.200	0.214	0.230	0.257	0.252	0.257	0.261	0.264
30		0.188	0.163	0.174	0.187	0.211	0.204	0.208	0.211	0.213
50		0.148	0.128	0.137	0.147	0.167	0.161	0.163	0.165	0.166
100		0.104	0.090	0.097	0.105	0.124	0.121	0.119	0.121	0.121
200		0.076	0.066	0.071	0.077	0.098	0.085	0.086	0.088	0.088
500		0.047	0.039	0.041	0.045	0.064	0.050	0.051	0.051	0.052
1000		0.032	0.031	0.029	0.032	0.050	0.036	0.037	0.039	0.037

COMPOUND (SEPARATED) POISSON PARTIAL DURATION MODEL WITH EXPONENTIAL DISTRIBUTIONS AS MARGINALS

TABLE A.65 BIAS OF SELECTED QUANTILES : (M1 = 1.0 M2 = 2.0 L1 = 2.0 L2 = 1.0)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	1.885	3.577	4.830	6.120	7.880	9.239	10.418	12.435	13.819
10		-0.005	0.027	0.041	0.046	0.046	0.044	0.061	0.040	0.038
20		-0.011	0.003	0.009	0.010	0.007	0.005	0.021	0.000	-0.001
30		-0.012	-0.003	0.003	0.004	0.003	0.001	0.018	-0.002	-0.002
50		-0.005	0.001	0.003	0.005	0.002	0.002	0.020	0.000	0.000
100		-0.001	0.001	0.001	0.002	0.001	0.000	0.017	-0.001	-0.001
200		0.001	0.001	0.002	0.002	0.002	0.002	0.019	0.001	0.001
500		-0.001	0.000	0.000	0.001	0.001	0.000	0.019	0.000	0.000
1000		-0.002	-0.002	-0.002	-0.002	-0.002	-0.002	0.016	-0.003	-0.002

TABLE A.66 RMSE OF SELECTED QUANTILES : (M1 = 1.0 M2 = 2.0 L1 = 2.0 L2 = 1.0)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	1.885	3.577	4.830	6.120	7.880	9.239	10.418	12.435	13.819
10		0.241	0.253	0.278	0.294	0.307	0.312	0.325	0.320	0.322
20		0.168	0.172	0.187	0.199	0.208	0.213	0.221	0.218	0.220
30		0.135	0.134	0.146	0.157	0.167	0.171	0.178	0.176	0.178
50		0.106	0.106	0.116	0.125	0.132	0.136	0.142	0.139	0.140
100		0.076	0.077	0.086	0.093	0.099	0.102	0.107	0.105	0.105
200		0.051	0.054	0.057	0.062	0.066	0.068	0.074	0.070	0.070
500		0.032	0.032	0.036	0.037	0.040	0.041	0.046	0.042	0.043
1000		0.025	0.025	0.029	0.030	0.034	0.032	0.037	0.033	0.033

COMPOUND (SEPARATED) POISSON PARTIAL DURATION MODEL WITH EXPONENTIAL DISTRIBUTIONS AS MARGINALS

TABLE A.67 BIAS OF SELECTED QUANTILES : (M1 = 1.0 M2 = 3.0 L1 = 0.7 L2 = 0.3)

SAMPLE SIZE	T = 2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q = 0.472	2.188	3.686	5.475	8.124	10.196	12.277	15.029	17.110
10	-0.039	0.025	0.064	0.070	0.063	0.059	0.055	0.054	0.053
20	-0.064	-0.011	0.021	0.028	0.018	0.015	0.012	0.011	0.012
30	-0.068	-0.025	-0.006	-0.001	-0.008	-0.010	-0.011	-0.011	-0.010
50	-0.025	-0.006	0.004	0.004	-0.005	-0.006	-0.007	-0.008	-0.007
100	-0.022	-0.012	-0.007	-0.005	-0.008	-0.009	-0.009	-0.008	-0.008
200	0.006	0.003	0.006	0.006	0.003	0.003	0.003	0.002	0.003
500	-0.004	-0.001	-0.002	-0.001	-0.003	-0.002	-0.002	-0.002	-0.001
1000	-0.004	-0.001	-0.001	0.000	-0.001	0.000	0.000	0.000	0.001

TABLE A.68 RMSE OF SELECTED QUANTILES : (M1 = 1.0 M2 = 3.0 L1 = 0.7 L2 = 0.3)

SAMPLE SIZE	T = 2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q = 0.472	2.188	3.686	5.475	8.124	10.196	12.277	15.029	17.110
10	0.973	0.501	0.527	0.540	0.555	0.565	0.573	0.581	0.586
20	0.686	0.347	0.380	0.409	0.426	0.431	0.435	0.439	0.441
30	0.557	0.269	0.289	0.316	0.332	0.337	0.341	0.344	0.345
50	0.434	0.202	0.219	0.241	0.254	0.259	0.261	0.262	0.262
100	0.300	0.141	0.148	0.166	0.176	0.178	0.178	0.177	0.177
200	0.214	0.102	0.114	0.131	0.139	0.139	0.139	0.139	0.138
500	0.138	0.065	0.072	0.082	0.088	0.088	0.087	0.088	0.087
1000	0.097	0.045	0.055	0.055	0.060	0.059	0.059	0.059	0.058

COMPOUND (SEPARATED) POISSON PARTIAL DURATION MODEL WITH EXPONENTIAL DISTRIBUTIONS AS MARGINALS

TABLE A.69 BIAS OF SELECTED QUANTILES : (M1 = 1.0 M2 = 3.0 L1 = 1.3 L2 = 0.7)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	1.551	3.834	5.796	7.870	10.640	12.732	14.818	17.571	19.652
10		-0.019	0.022	0.026	0.022	0.019	0.019	0.018	0.019	0.020
20		-0.025	-0.003	0.000	-0.002	-0.004	-0.003	-0.003	-0.002	-0.001
30		-0.010	0.001	0.001	-0.001	-0.004	-0.003	-0.002	-0.001	0.000
50		-0.004	0.002	-0.001	-0.003	-0.006	-0.006	-0.006	-0.005	-0.005
100		-0.001	0.000	0.002	0.000	-0.001	0.000	0.000	0.001	0.002
200		0.003	0.001	0.000	0.000	-0.001	0.000	0.000	0.000	0.000
500		-0.001	-0.001	-0.001	0.000	-0.001	0.000	0.001	0.000	0.002
1000		-0.002	-0.004	-0.004	-0.003	-0.003	-0.002	-0.002	-0.001	-0.001

TABLE A.70 RMSE OF SELECTED QUANTILES : (M1 = 1.0 M2 = 3.0 L1 = 1.3 L2 = 0.7)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	1.551	3.834	5.796	7.870	10.640	12.732	14.818	17.571	19.652
10		0.369	0.367	0.382	0.392	0.398	0.402	0.406	0.409	0.411
20		0.251	0.244	0.261	0.268	0.271	0.273	0.275	0.276	0.277
30		0.202	0.197	0.211	0.217	0.220	0.221	0.221	0.221	0.221
50		0.160	0.158	0.170	0.174	0.174	0.173	0.173	0.171	0.171
100		0.113	0.111	0.120	0.124	0.126	0.124	0.124	0.124	0.123
200		0.082	0.082	0.090	0.091	0.091	0.090	0.090	0.089	0.089
500		0.050	0.050	0.055	0.056	0.054	0.053	0.052	0.077	0.052
1000		0.035	0.045	0.039	0.042	0.039	0.038	0.039	0.037	0.037

COMPOUND (SEPARATED) POISSON PARTIAL DURATION MODEL WITH EXPONENTIAL DISTRIBUTIONS AS MARGINALS

TABLE A.71 BIAS OF SELECTED QUANTILES : (M1 = 1.0 M2 = 3.0 L1 = 2.0 L2 = 1.0)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	2.223	4.744	6.814	8.926	11.708	13.801	15.878	18.641	20.722
10		0.007	0.033	0.031	0.026	0.024	0.023	0.023	0.023	0.023
20		-0.006	0.004	-0.001	-0.005	-0.008	-0.008	-0.007	-0.009	-0.008
30		-0.011	-0.003	-0.005	-0.007	-0.008	-0.006	-0.005	-0.005	-0.005
50		-0.002	0.000	-0.001	-0.002	-0.003	-0.002	-0.001	-0.001	-0.001
100		-0.001	-0.002	-0.003	-0.003	-0.003	-0.002	-0.001	-0.002	-0.001
200		0.002	0.000	0.000	0.000	-0.001	0.001	0.001	0.001	0.001
500		-0.001	-0.002	-0.002	-0.001	-0.001	0.000	0.001	0.001	0.001
1000		-0.002	-0.004	-0.004	-0.004	-0.004	-0.003	-0.002	-0.002	-0.002

TABLE A.72 RMSE OF SELECTED QUANTILES : (M1 = 1.0 M2 = 3.0 L1 = 2.0 L2 = 1.0)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	2.223	4.744	6.814	8.926	11.708	13.801	15.878	18.641	20.722
10		0.277	0.314	0.327	0.331	0.334	0.335	0.337	0.337	0.338
20		0.193	0.218	0.228	0.230	0.232	0.231	0.231	0.232	0.230
30		0.152	0.170	0.180	0.184	0.184	0.184	0.184	0.183	0.182
50		0.119	0.134	0.141	0.144	0.145	0.143	0.143	0.143	0.142
100		0.085	0.103	0.107	0.108	0.108	0.108	0.107	0.107	0.106
200		0.057	0.069	0.071	0.072	0.071	0.071	0.071	0.070	0.070
500		0.036	0.042	0.046	0.046	0.044	0.042	0.042	0.042	0.042
1000		0.028	0.036	0.035	0.036	0.037	0.034	0.033	0.033	0.033

SIMPLE (MIXED) POISSON PARTIAL DURATION MODEL WITH EXPONENTIAL DISTRIBUTION AS MARGINAL

TABLE A.73 BIAS OF SELECTED QUANTILES : (M1 = 1.0 M2 = 1.5 L1 = 0.7 L2 = 0.3)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	0.409	1.707	2.596	3.474	4.650	5.562	6.494	7.757	8.734
10		-0.063	0.001	-0.004	-0.014	-0.029	-0.041	-0.054	-0.070	-0.081
20		-0.028	0.000	-0.008	-0.020	-0.036	-0.049	-0.062	-0.078	-0.089
30		-0.029	-0.008	-0.018	-0.029	-0.046	-0.059	-0.072	-0.088	-0.099
50		0.006	0.007	-0.005	-0.018	-0.035	-0.049	-0.062	-0.078	-0.090
100		0.011	0.002	-0.010	-0.024	-0.041	-0.055	-0.068	-0.084	-0.096
200		0.038	0.014	0.000	-0.014	-0.032	-0.046	-0.060	-0.077	-0.088
500		0.026	0.011	-0.002	-0.016	-0.034	-0.048	-0.061	-0.078	-0.089
1000		0.026	0.010	-0.003	-0.016	-0.035	-0.048	-0.062	-0.078	-0.090

TABLE A.74 RMSE OF SELECTED QUANTILES : (M1 = 1.0 M2 = 1.5 L1 = 0.7 L2 = 0.3)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	0.409	1.707	2.596	3.474	4.650	5.562	6.494	7.757	8.734
10		1.103	0.401	0.357	0.340	0.329	0.324	0.320	0.317	0.316
20		0.699	0.288	0.262	0.252	0.245	0.243	0.241	0.242	0.243
30		0.564	0.228	0.207	0.199	0.195	0.194	0.196	0.198	0.201
50		0.434	0.172	0.155	0.149	0.147	0.148	0.150	0.155	0.160
100		0.299	0.122	0.110	0.107	0.109	0.113	0.118	0.127	0.134
200		0.216	0.091	0.082	0.080	0.082	0.087	0.094	0.105	0.113
500		0.141	0.059	0.052	0.052	0.058	0.067	0.076	0.090	0.100
1000		0.099	0.040	0.035	0.037	0.047	0.058	0.069	0.084	0.095

SIMPLE (MIXED) POISSON PARTIAL DURATION MODEL WITH EXPONENTIAL DISTRIBUTION AS MARGINAL

TABLE A.75 BIAS OF SELECTED QUANTILES : (M1 = 1.0 M2 = 1.5 L1 = 1.3 L2 = 0.7)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	1.222	2.584	3.521	4.448	5.687	6.642	6.579	8.921	9.924
10		0.011	-0.003	-0.016	-0.030	-0.047	-0.060	0.074	-0.086	-0.096
20		0.001	-0.013	-0.026	-0.039	-0.056	-0.069	0.064	-0.095	-0.104
30		0.015	-0.004	-0.018	-0.032	-0.050	-0.063	0.071	-0.089	-0.099
50		0.017	-0.005	-0.020	-0.034	-0.053	-0.066	0.067	-0.092	-0.102
100		0.020	0.000	-0.015	-0.030	-0.048	-0.061	0.073	-0.088	-0.097
200		0.022	-0.001	-0.016	-0.031	-0.049	-0.062	0.071	-0.089	-0.099
500		0.019	-0.002	-0.017	-0.032	-0.050	-0.063	0.070	-0.090	-0.099
1000		0.018	-0.004	-0.018	-0.033	-0.051	-0.064	0.069	-0.091	-0.101

TABLE A.76 RMSE OF SELECTED QUANTILES : (M1 = 1.0 M2 = 1.5 L1 = 1.3 L2 = 0.7)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	1.222	2.584	3.521	4.448	5.687	6.642	6.579	8.921	9.924
10		0.324	0.255	0.243	0.238	0.234	0.233	0.268	0.235	0.236
20		0.224	0.177	0.170	0.168	0.168	0.170	0.189	0.178	0.182
30		0.186	0.147	0.141	0.139	0.141	0.144	0.163	0.153	0.158
50		0.147	0.114	0.109	0.109	0.113	0.118	0.130	0.132	0.138
100		0.105	0.081	0.079	0.080	0.087	0.094	0.109	0.111	0.119
200		0.078	0.059	0.058	0.062	0.072	0.081	0.092	0.102	0.110
500		0.049	0.035	0.038	0.045	0.059	0.070	0.079	0.094	0.104
1000		0.036	0.025	0.030	0.040	0.056	0.068	0.073	0.093	0.103

SIMPLE (MIXED) POISSON PARTIAL DURATION MODEL WITH EXPONENTIAL DISTRIBUTION AS MARGINAL

TABLE A.77 BIAS OF SELECTED QUANTILES : (M1 = 1.0 M2 = 1.5 L1 = 2.0 L2 = 1.0)

SAMPLE SIZE	T = 2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q = 1.690	3.064	4.009	4.944	6.192	7.154	8.131	9.444	10.450
10	0.016	-0.004	-0.018	-0.032	-0.050	-0.063	-0.074	-0.089	-0.098
20	0.004	-0.014	-0.028	-0.042	-0.059	-0.072	-0.083	-0.097	-0.107
30	0.004	-0.016	-0.030	-0.044	-0.062	-0.075	-0.086	-0.100	-0.110
50	0.009	-0.014	-0.029	-0.043	-0.061	-0.074	-0.086	-0.100	-0.109
100	0.011	-0.010	-0.025	-0.039	-0.057	-0.070	-0.082	-0.096	-0.106
200	0.013	-0.009	-0.024	-0.038	-0.057	-0.069	-0.081	-0.095	-0.105
500	0.011	-0.011	-0.025	-0.040	-0.058	-0.071	-0.082	-0.097	-0.106
1000	0.010	-0.011	-0.026	-0.041	-0.059	-0.071	-0.083	-0.097	-0.107

TABLE A.78 RMSE OF SELECTED QUANTILES : (M1 = 1.0 M2 = 1.5 L1 = 2.0 L2 = 1.0)

SAMPLE SIZE	T = 2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q = 1.690	3.064	4.009	4.944	6.192	7.154	8.131	9.444	10.450
10	0.233	0.201	0.194	0.191	0.189	0.190	0.192	0.195	0.197
20	0.161	0.141	0.138	0.138	0.142	0.145	0.150	0.156	0.161
30	0.129	0.112	0.111	0.113	0.118	0.124	0.130	0.138	0.145
50	0.103	0.091	0.091	0.094	0.102	0.109	0.116	0.126	0.133
100	0.074	0.065	0.067	0.072	0.082	0.091	0.100	0.111	0.119
200	0.051	0.044	0.048	0.056	0.069	0.080	0.090	0.103	0.111
500	0.033	0.029	0.036	0.047	0.063	0.075	0.086	0.099	0.109
1000	0.026	0.024	0.033	0.045	0.062	0.074	0.085	0.099	0.108

SIMPLE (MIXED) POISSON PARTIAL DURATION MODEL WITH EXPONENTIAL DISTRIBUTION AS MARGINAL

TABLE A.79 BIAS OF SELECTED QUANTILES : (M1 = 1.0 M2 = 2.0 L1 = 0.7 L2 = 0.3)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	0.437	1.885	2.949	4.066	5.655	6.933	8.257	10.050	11.422
10		-0.008	0.028	-0.005	-0.043	-0.093	-0.127	-0.155	-0.184	-0.202
20		0.032	0.026	-0.011	-0.051	-0.102	-0.136	-0.164	-0.194	-0.211
30		0.028	0.014	-0.023	-0.063	-0.114	-0.147	-0.175	-0.205	-0.222
50		0.064	0.030	-0.010	-0.052	-0.104	-0.138	-0.166	-0.196	-0.214
100		0.069	0.025	-0.016	-0.058	-0.109	-0.144	-0.172	-0.202	-0.219
200		0.099	0.038	-0.005	-0.047	-0.100	-0.135	-0.164	-0.194	-0.212
500		0.085	0.034	-0.008	-0.050	-0.102	-0.137	-0.166	-0.196	-0.213
1000		0.086	0.034	-0.008	-0.050	-0.103	-0.137	-0.166	-0.196	-0.213

TABLE A.80 RMSE OF SELECTED QUANTILES : (M1 = 1.0 M2 = 2.0 L1 = 0.7 L2 = 0.3)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	0.437	1.885	2.949	4.066	5.655	6.933	8.257	10.050	11.422
10		1.206	0.436	0.381	0.357	0.343	0.340	0.342	0.347	0.351
20		0.750	0.314	0.280	0.266	0.264	0.270	0.278	0.290	0.299
30		0.604	0.247	0.220	0.213	0.221	0.233	0.247	0.264	0.275
50		0.467	0.188	0.165	0.162	0.176	0.194	0.212	0.233	0.247
100		0.326	0.133	0.116	0.121	0.148	0.172	0.195	0.220	0.235
200		0.248	0.104	0.087	0.094	0.125	0.153	0.178	0.205	0.222
500		0.170	0.071	0.055	0.071	0.113	0.144	0.171	0.200	0.217
1000		0.133	0.054	0.038	0.061	0.107	0.140	0.168	0.198	0.215

SIMPLE (MIXED) POISSON PARTIAL DURATION MODEL WITH EXPONENTIAL DISTRIBUTION AS MARGINAL

TABLE A.81 BIAS OF SELECTED QUANTILES : (M1 = 1.0 M2 = 2.0 L1 = 1.3 L2 = 0.7)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	1.351	2.984	4.198	5.456	6.939	8.538	9.904	11.724	13.106
10		0.052	-0.007	-0.050	-0.090	-0.101	-0.158	-0.179	-0.200	-0.212
20		0.041	-0.017	-0.060	-0.099	-0.111	-0.167	-0.187	-0.208	-0.220
30		0.054	-0.010	-0.054	-0.094	-0.106	-0.163	-0.183	-0.204	-0.217
50		0.057	-0.010	-0.055	-0.096	-0.108	-0.165	-0.185	-0.206	-0.219
100		0.060	-0.006	-0.051	-0.092	-0.104	-0.161	-0.182	-0.203	-0.215
200		0.062	-0.006	-0.052	-0.092	-0.105	-0.162	-0.183	-0.204	-0.216
500		0.059	-0.008	-0.053	-0.093	-0.105	-0.162	-0.183	-0.204	-0.216
1000		0.058	-0.009	-0.054	-0.094	-0.107	-0.164	-0.184	-0.205	-0.218

TABLE A.82 RMSE OF SELECTED QUANTILES : (M1 = 1.0 M2 = 2.0 L1 = 1.3 L2 = 0.7)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	1.351	2.984	4.198	5.456	6.939	8.538	9.904	11.724	13.106
10		0.357	0.272	0.257	0.254	0.254	0.269	0.277	0.287	0.293
20		0.245	0.188	0.183	0.191	0.195	0.224	0.237	0.251	0.260
30		0.206	0.155	0.153	0.164	0.169	0.204	0.219	0.235	0.245
50		0.168	0.121	0.124	0.141	0.148	0.190	0.207	0.225	0.236
100		0.126	0.086	0.094	0.118	0.127	0.175	0.193	0.213	0.224
200		0.103	0.063	0.078	0.107	0.118	0.169	0.189	0.209	0.221
500		0.076	0.038	0.063	0.098	0.110	0.165	0.185	0.206	0.218
1000		0.067	0.027	0.059	0.097	0.109	0.165	0.185	0.206	0.219

SIMPLE (MIXED) POISSON PARTIAL DURATION MODEL WITH EXPONENTIAL DISTRIBUTION AS MARGINAL

TABLE A.83 BIAS OF SELECTED QUANTILES : (M1 = 1.0 M2 = 2.0 L1 = 2.0 L2 = 1.0)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	1.885	3.577	4.830	6.120	7.880	9.239	10.418	12.435	13.819
10		0.043	-0.022	-0.066	-0.104	-0.145	-0.168	-0.172	-0.207	-0.219
20		0.029	-0.034	-0.078	-0.115	-0.155	-0.178	-0.182	-0.216	-0.228
30		0.028	-0.037	-0.081	-0.118	-0.158	-0.182	-0.186	-0.220	-0.231
50		0.034	-0.034	-0.078	-0.116	-0.156	-0.180	-0.184	-0.218	-0.230
100		0.036	-0.031	-0.075	-0.113	-0.154	-0.177	-0.181	-0.216	-0.227
200		0.038	-0.030	-0.074	-0.112	-0.153	-0.177	-0.181	-0.215	-0.227
500		0.036	-0.031	-0.076	-0.114	-0.154	-0.178	-0.182	-0.216	-0.227
1000		0.035	-0.032	-0.076	-0.114	-0.155	-0.178	-0.182	-0.217	-0.228

TABLE A.84 RMSE OF SELECTED QUANTILES : (M1 = 1.0 M2 = 2.0 L1 = 2.0 L2 = 1.0)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	1.885	3.577	4.830	6.120	7.880	9.239	10.418	12.435	13.819
10		0.256	0.213	0.208	0.214	0.229	0.241	0.242	0.263	0.271
20		0.177	0.153	0.159	0.175	0.199	0.216	0.219	0.245	0.255
30		0.142	0.122	0.135	0.157	0.186	0.205	0.208	0.237	0.247
50		0.116	0.099	0.117	0.143	0.175	0.195	0.199	0.230	0.241
100		0.088	0.075	0.099	0.129	0.164	0.186	0.190	0.222	0.233
200		0.066	0.054	0.085	0.119	0.158	0.180	0.184	0.218	0.229
500		0.049	0.042	0.080	0.116	0.156	0.179	0.183	0.217	0.228
1000		0.043	0.039	0.079	0.116	0.156	0.179	0.183	0.217	0.229

SIMPLE (MIXED) POISSON PARTIAL DURATION MODEL WITH EXPONENTIAL DISTRIBUTION AS MARGINAL

TABLE A.85 BIAS OF SELECTED QUANTILES : (M1 = 1.0 M2 = 3.0 L1 = 0.7 L2 = 0.3)

SAMPLE SIZE	T = 2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q = 0.472	2.188	3.686	5.475	8.124	10.196	12.277	15.029	17.110
10	0.132	0.096	-0.014	-0.120	-0.218	-0.265	-0.296	-0.325	-0.340
20	0.181	0.091	-0.024	-0.130	-0.229	-0.275	-0.306	-0.335	-0.350
30	0.173	0.074	-0.040	-0.145	-0.242	-0.287	-0.318	-0.346	-0.362
50	0.214	0.091	-0.026	-0.134	-0.233	-0.279	-0.310	-0.339	-0.354
100	0.219	0.086	-0.032	-0.140	-0.238	-0.284	-0.315	-0.344	-0.359
200	0.254	0.101	-0.020	-0.129	-0.229	-0.276	-0.308	-0.337	-0.352
500	0.238	0.096	-0.024	-0.132	-0.232	-0.278	-0.310	-0.338	-0.354
1000	0.239	0.097	-0.023	-0.132	-0.231	-0.278	-0.309	-0.338	-0.354

TABLE A.86 RMSE OF SELECTED QUANTILES : (M1 = 1.0 M2 = 3.0 L1 = 0.7 L2 = 0.3)

SAMPLE SIZE	T = 2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q = 0.472	2.188	3.686	5.475	8.124	10.196	12.277	15.029	17.110
10	1.466	0.526	0.429	0.392	0.394	0.406	0.417	0.430	0.437
20	0.898	0.384	0.315	0.303	0.332	0.355	0.374	0.393	0.404
30	0.727	0.300	0.246	0.256	0.304	0.335	0.358	0.380	0.393
50	0.581	0.238	0.187	0.209	0.272	0.309	0.335	0.360	0.373
100	0.430	0.175	0.131	0.178	0.257	0.298	0.327	0.353	0.368
200	0.368	0.153	0.099	0.154	0.241	0.284	0.315	0.342	0.358
500	0.293	0.120	0.065	0.142	0.236	0.282	0.313	0.341	0.356
1000	0.267	0.109	0.047	0.137	0.233	0.279	0.311	0.339	0.355

SIMPLE (MIXED) POISSON PARTIAL DURATION MODEL WITH EXPONENTIAL DISTRIBUTION AS MARGINAL

TABLE A.87 BIAS OF SELECTED QUANTILES : (M1 = 1.0 M2 = 3.0 L1 = 1.3 L2 = 0.7)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	1.551	3.834	5.796	7.870	10.640	12.732	14.818	17.571	19.652
10		0.157	-0.024	-0.132	-0.203	-0.260	-0.287	-0.307	-0.326	-0.337
20		0.142	-0.036	-0.142	-0.213	-0.269	-0.296	-0.315	-0.334	-0.344
30		0.155	-0.030	-0.138	-0.210	-0.267	-0.294	-0.313	-0.332	-0.343
50		0.159	-0.030	-0.139	-0.210	-0.267	-0.295	-0.314	-0.333	-0.344
100		0.162	-0.026	-0.135	-0.207	-0.265	-0.292	-0.312	-0.331	-0.341
200		0.165	-0.026	-0.136	-0.208	-0.265	-0.293	-0.312	-0.331	-0.342
500		0.161	-0.027	-0.136	-0.208	-0.265	-0.293	-0.312	-0.331	-0.342
1000		0.159	-0.029	-0.138	-0.210	-0.267	-0.294	-0.314	-0.332	-0.343

TABLE A.88 RMSE OF SELECTED QUANTILES : (M1 = 1.0 M2 = 3.0 L1 = 1.3 L2 = 0.7)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	1.551	3.834	5.796	7.870	10.640	12.732	14.818	17.571	19.652
10		0.452	0.303	0.293	0.313	0.340	0.356	0.369	0.382	0.389
20		0.320	0.209	0.228	0.268	0.308	0.329	0.345	0.360	0.369
30		0.281	0.171	0.201	0.248	0.293	0.316	0.333	0.350	0.360
50		0.246	0.136	0.180	0.234	0.284	0.308	0.327	0.344	0.354
100		0.208	0.097	0.158	0.220	0.273	0.299	0.318	0.336	0.346
200		0.191	0.074	0.148	0.215	0.270	0.296	0.316	0.334	0.345
500		0.171	0.048	0.140	0.210	0.267	0.294	0.313	0.332	0.343
1000		0.164	0.041	0.140	0.211	0.267	0.295	0.314	0.333	0.344

SIMPLE (MIXED) POISSON PARTIAL DURATION MODEL WITH EXPONENTIAL DISTRIBUTION AS MARGINAL

TABLE A.89 BIAS OF SELECTED QUANTILES : (M1 = 1.0 M2 = 3.0 L1 = 2.0 L2 = 1.0)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	2.223	4.744	6.814	8.926	11.708	13.801	15.878	18.641	20.722
10		0.110	-0.075	-0.170	-0.229	-0.278	-0.302	-0.319	-0.336	-0.346
20		0.092	-0.090	-0.182	-0.241	-0.289	-0.312	-0.329	-0.346	-0.356
30		0.089	-0.093	-0.186	-0.245	-0.292	-0.316	-0.333	-0.350	-0.360
50		0.097	-0.089	-0.183	-0.242	-0.290	-0.313	-0.330	-0.348	-0.358
100		0.097	-0.088	-0.181	-0.241	-0.289	-0.312	-0.329	-0.347	-0.356
200		0.100	-0.086	-0.180	-0.240	-0.288	-0.311	-0.328	-0.346	-0.356
500		0.098	-0.087	-0.181	-0.240	-0.288	-0.312	-0.329	-0.346	-0.356
1000		0.096	-0.088	-0.182	-0.241	-0.289	-0.313	-0.330	-0.347	-0.357

TABLE A.90 RMSE OF SELECTED QUANTILES : (M1 = 1.0 M2 = 3.0 L1 = 2.0 L2 = 1.0)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	2.223	4.744	6.814	8.926	11.708	13.801	15.878	18.641	20.722
10		0.317	0.239	0.263	0.294	0.327	0.344	0.357	0.371	0.379
20		0.228	0.184	0.231	0.274	0.313	0.334	0.348	0.364	0.373
30		0.186	0.156	0.216	0.265	0.307	0.328	0.344	0.360	0.369
50		0.162	0.133	0.203	0.255	0.299	0.322	0.338	0.354	0.364
100		0.136	0.114	0.193	0.248	0.294	0.317	0.333	0.350	0.360
200		0.118	0.099	0.185	0.243	0.290	0.313	0.330	0.347	0.357
500		0.105	0.092	0.183	0.242	0.289	0.313	0.329	0.347	0.357
1000		0.101	0.091	0.183	0.242	0.290	0.313	0.330	0.347	0.357

COMPOUND (SEPARATED) POISSON PARTIAL DURATION MODEL WITH WEIBULL DISTRIBUTIONS AS MARGINALS

TABLE A.91 BIAS OF SELECTED QUANTILES: (M1 = 1.0 M2 = 1.5 CV1 = 0.5 CV2 = 1.0 L1 = 1.0 L2 = 0.5)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	1.027	1.791	2.433	3.417	4.813	5.861	6.904	8.281	9.321
10		-0.006	0.031	0.048	-0.019	-0.069	-0.085	-0.092	-0.094	-0.093
20		-0.018	-0.004	0.020	-0.019	-0.043	-0.048	-0.048	-0.045	-0.042
30		-0.012	-0.002	0.013	-0.022	-0.040	-0.042	-0.042	-0.041	-0.039
50		-0.003	-0.003	0.009	-0.013	-0.023	-0.024	-0.023	-0.022	-0.021
100		-0.002	-0.001	0.007	-0.007	-0.010	-0.010	-0.008	-0.007	-0.006
200		0.003	0.003	0.009	0.002	0.000	0.001	0.001	0.002	0.002
500		-0.001	0.000	0.002	-0.002	-0.002	-0.002	-0.002	-0.002	-0.002
1000		-0.001	-0.001	0.000	-0.001	-0.001	-0.001	-0.001	-0.001	0.000

TABLE A.92 RMSE OF SELECTED QUANTILES: (M1 = 1.0 M2 = 1.5 CV1 = 0.5 CV2 = 1.0 L1 = 1.0 L2 = 0.5)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	1.027	1.791	2.433	3.417	4.813	5.861	6.904	8.281	9.321
10		0.268	0.290	0.392	0.436	0.500	0.553	0.607	0.681	0.738
20		0.193	0.178	0.269	0.322	0.370	0.403	0.434	0.475	0.505
30		0.155	0.143	0.222	0.266	0.300	0.323	0.346	0.374	0.395
50		0.113	0.102	0.167	0.207	0.233	0.251	0.268	0.289	0.305
100		0.082	0.073	0.121	0.154	0.170	0.182	0.194	0.209	0.221
200		0.056	0.049	0.083	0.109	0.118	0.125	0.132	0.142	0.148
500		0.037	0.031	0.053	0.071	0.077	0.082	0.087	0.094	0.098
1000		0.026	0.022	0.036	0.048	0.052	0.056	0.059	0.064	0.067

COMPOUND (SEPARATED) POISSON PARTIAL DURATION MODEL WITH WEIBULL DISTRIBUTIONS AS MARGINALS

TABLE A.93 BIAS OF SELECTED QUANTILES: (M1 = 1.0 M2 = 1.5 CV1 = 0.5 CV2 = 1.0 L1 = 1.3 L2 = 0.7)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	1.238	2.029	2.857	3.920	5.318	6.365	7.409	8.785	9.826
10		-0.011	0.034	0.012	-0.037	-0.061	-0.067	-0.068	-0.065	-0.060
20		-0.014	0.006	-0.001	-0.027	-0.032	-0.030	-0.028	-0.023	-0.019
30		-0.003	0.005	-0.001	-0.025	-0.033	-0.035	-0.035	-0.035	-0.034
50		-0.002	0.005	0.001	-0.016	-0.020	-0.020	-0.020	-0.019	-0.018
100		-0.001	0.004	0.004	-0.003	-0.001	0.000	0.001	0.003	0.004
200		0.002	0.002	0.003	-0.002	-0.003	-0.003	-0.003	-0.002	-0.002
500		0.000	0.001	0.001	0.000	0.000	0.001	0.001	0.001	0.001
1000		-0.001	-0.001	-0.002	-0.003	-0.003	-0.003	-0.003	-0.003	-0.003

TABLE A.94 RMSE OF SELECTED QUANTILES: (M1 = 1.0 M2 = 1.5 CV1 = 0.5 CV2 = 1.0 L1 = 1.3 L2 = 0.7)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	1.238	2.029	2.857	3.920	5.318	6.365	7.409	8.785	9.826
10		0.201	0.294	0.359	0.394	0.449	0.493	0.538	0.598	0.643
20		0.136	0.183	0.253	0.286	0.321	0.347	0.373	0.408	0.433
30		0.109	0.141	0.207	0.236	0.261	0.278	0.295	0.317	0.332
50		0.085	0.112	0.170	0.192	0.209	0.222	0.235	0.251	0.263
100		0.059	0.076	0.122	0.137	0.148	0.158	0.167	0.178	0.186
200		0.043	0.055	0.092	0.103	0.111	0.117	0.124	0.132	0.137
500		0.027	0.031	0.053	0.060	0.065	0.069	0.074	0.079	0.083
1000		0.019	0.022	0.038	0.042	0.045	0.047	0.050	0.054	0.056

COMPOUND (SEPARATED) POISSON PARTIAL DURATION MODEL WITH WEIBULL DISTRIBUTIONS AS MARGINALS

TABLE A.95 BIAS OF SELECTED QUANTILES: (M1 = 1.0 M2 = 1.5 CV1 = 0.5 CV2 = 1.0 L1 = 2.0 L2 = 1.0)

SAMPLE SIZE	T = 2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q = 1.500	2.371	3.377	4.455	5.853	6.900	7.944	9.320	10.361
10	-0.002	0.042	0.004	-0.019	-0.026	-0.025	-0.023	-0.017	-0.012
20	-0.005	0.014	-0.014	-0.027	-0.030	-0.030	-0.028	-0.026	-0.024
30	-0.007	0.004	-0.015	-0.024	-0.026	-0.026	-0.025	-0.024	-0.024
50	-0.002	0.007	-0.006	-0.012	-0.013	-0.013	-0.013	-0.012	-0.011
100	-0.001	0.003	-0.004	-0.006	-0.005	-0.005	-0.004	-0.003	-0.003
200	0.001	0.004	-0.001	-0.002	-0.002	-0.002	-0.002	-0.002	-0.002
500	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.001	0.001
1000	-0.001	-0.001	-0.003	-0.002	-0.002	-0.002	-0.002	-0.002	-0.002

TABLE A.96 RMSE OF SELECTED QUANTILES: (M1 = 1.0 M2 = 1.5 CV1 = 0.5 CV2 = 1.0 L1 = 2.0 L2 = 1.0)

SAMPLE SIZE	T = 2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q = 1.500	2.371	3.377	4.455	5.853	6.900	7.944	9.320	10.361
10	0.151	0.282	0.331	0.365	0.412	0.450	0.488	0.540	0.580
20	0.105	0.191	0.235	0.257	0.283	0.302	0.320	0.343	0.359
30	0.082	0.146	0.189	0.206	0.224	0.238	0.250	0.266	0.278
50	0.065	0.116	0.154	0.168	0.184	0.196	0.208	0.223	0.233
100	0.046	0.084	0.115	0.123	0.133	0.141	0.148	0.157	0.164
200	0.031	0.056	0.078	0.084	0.091	0.097	0.102	0.109	0.113
500	0.019	0.033	0.047	0.050	0.055	0.058	0.062	0.066	0.069
1000	0.015	0.026	0.037	0.039	0.042	0.044	0.047	0.050	0.052

COMPOUND (SEPARATED) POISSON PARTIAL DURATION MODEL WITH WEIBULL DISTRIBUTIONS AS MARGINALS

TABLE A.97 BIAS OF SELECTED QUANTILES: (M1 = 1.0 M2 = 1.5 CV1= 0.5 CV2= 1.5 L1 = 1.0 L2 = 0.5)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	0.966	1.722	2.384	3.857	6.365	8.486	10.780	14.058	16.711
10		-0.016	0.044	0.142	0.045	-0.018	-0.027	-0.020	0.001	0.023
20		-0.024	-0.003	0.071	0.014	-0.016	-0.016	-0.008	0.006	0.019
30		-0.016	-0.002	0.051	-0.005	-0.028	-0.027	-0.022	-0.013	-0.005
50		-0.005	-0.004	0.033	-0.004	-0.016	-0.014	-0.010	-0.004	0.001
100		-0.002	-0.001	0.021	-0.003	-0.006	-0.003	0.000	0.004	0.008
200		0.003	0.004	0.018	0.006	0.005	0.006	0.007	0.009	0.011
500		-0.001	0.000	0.005	-0.001	-0.001	-0.001	0.000	0.000	0.001
1000		-0.001	-0.001	0.001	-0.001	-0.001	0.000	0.000	0.001	0.001

TABLE A.98 RMSE OF SELECTED QUANTILES: (M1 = 1.0 M2 = 1.5 CV1= 0.5 CV2= 1.5 L1 = 1.0 L2 = 0.5)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	0.966	1.722	2.384	3.857	6.365	8.486	10.780	14.058	16.711
10		0.294	0.352	0.612	0.694	0.811	0.927	1.061	1.261	1.429
20		0.212	0.191	0.388	0.485	0.561	0.619	0.680	0.762	0.827
30		0.169	0.149	0.313	0.393	0.443	0.480	0.519	0.570	0.608
50		0.123	0.104	0.227	0.305	0.343	0.370	0.398	0.435	0.463
100		0.088	0.073	0.159	0.227	0.250	0.269	0.289	0.315	0.334
200		0.060	0.049	0.107	0.162	0.174	0.184	0.196	0.210	0.220
500		0.040	0.032	0.065	0.104	0.113	0.120	0.128	0.138	0.144
1000		0.028	0.022	0.044	0.071	0.076	0.081	0.087	0.093	0.098

COMPOUND (SEPARATED) POISSON PARTIAL DURATION MODEL WITH WEIBULL DISTRIBUTIONS AS MARGINALS

TABLE A.99 BIAS OF SELECTED QUANTILES: (M1 = 1.0 M2 = 1.5 CV1= 0.5 CV2= 1.5 L1 = 1.3 L2 = 0.7)

SAMPLE SIZE	T = 2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q = 1.175	1.951	2.961	4.717	7.363	9.574	11.950	15.327	18.049
10	-0.017	0.069	0.084	0.002	-0.023	-0.019	-0.008	0.015	0.036
20	-0.017	0.017	0.036	-0.011	-0.012	-0.005	0.005	0.020	0.032
30	-0.004	0.010	0.025	-0.018	-0.025	-0.025	-0.022	-0.017	-0.013
50	-0.003	0.008	0.020	-0.012	-0.014	-0.013	-0.011	-0.007	-0.004
100	-0.001	0.006	0.015	0.002	0.005	0.008	0.011	0.015	0.018
200	0.002	0.003	0.009	0.000	0.000	0.001	0.001	0.002	0.003
500	-0.001	0.001	0.004	0.001	0.002	0.002	0.003	0.004	0.004
1000	-0.001	-0.001	-0.002	-0.004	-0.004	-0.004	-0.004	-0.004	-0.004

TABLE A.100 RMSE OF SELECTED QUANTILES: (M1 = 1.0 M2 = 1.5 CV1= 0.5 CV2= 1.5 L1 = 1.3 L2 = 0.7)

SAMPLE SIZE	T = 2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q = 1.175	1.951	2.961	4.717	7.363	9.574	11.950	15.327	18.049
10	0.211	0.390	0.555	0.601	0.699	0.789	0.891	1.039	1.159
20	0.143	0.216	0.373	0.423	0.480	0.527	0.576	0.644	0.698
30	0.115	0.160	0.302	0.346	0.383	0.411	0.439	0.475	0.503
50	0.090	0.121	0.246	0.281	0.307	0.327	0.348	0.376	0.397
100	0.063	0.081	0.178	0.203	0.219	0.234	0.248	0.267	0.280
200	0.046	0.058	0.134	0.151	0.162	0.172	0.182	0.194	0.202
500	0.028	0.032	0.078	0.087	0.095	0.102	0.108	0.116	0.122
1000	0.019	0.023	0.055	0.061	0.065	0.069	0.073	0.078	0.082

COMPOUND (SEPARATED) POISSON PARTIAL DURATION MODEL WITH WEIBULL DISTRIBUTIONS AS MARGINALS

TABLE A.101 BIAS OF SELECTED QUANTILES: (M1 = 1.0 M2 = 1.5 CV1= 0.5 CV2= 1.5 L1 = 2.0 L2 = 1.0)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	1.438	2.312	3.792	5.686	8.470	10.771	13.231	16.709	19.501
10		-0.006	0.099	0.047	0.016	0.017	0.027	0.041	0.063	0.081
20		-0.007	0.043	0.000	-0.018	-0.017	-0.013	-0.007	0.001	0.008
30		-0.007	0.021	-0.009	-0.021	-0.021	-0.019	-0.016	-0.012	-0.009
50		-0.003	0.019	-0.002	-0.008	-0.008	-0.006	-0.004	-0.001	0.001
100		-0.001	0.010	-0.003	-0.004	-0.002	0.000	0.001	0.004	0.005
200		0.001	0.007	0.000	-0.001	0.000	0.000	0.000	0.001	0.001
500		0.000	0.002	0.000	0.000	0.001	0.002	0.002	0.003	0.003
1000		-0.001	-0.001	-0.003	-0.003	-0.003	-0.003	-0.002	-0.002	-0.002

TABLE A.102 RMSE OF SELECTED QUANTILES: (M1 = 1.0 M2 = 1.5 CV1= 0.5 CV2= 1.5 L1 = 2.0 L2 = 1.0)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	1.438	2.312	3.792	5.686	8.470	10.771	13.231	16.709	19.501
10		0.154	0.416	0.510	0.565	0.655	0.735	0.827	0.964	1.081
20		0.106	0.264	0.349	0.381	0.422	0.453	0.485	0.527	0.558
30		0.083	0.191	0.276	0.301	0.329	0.350	0.371	0.398	0.417
50		0.065	0.148	0.226	0.246	0.271	0.290	0.309	0.332	0.349
100		0.046	0.104	0.170	0.180	0.195	0.207	0.218	0.233	0.243
200		0.031	0.067	0.115	0.122	0.133	0.142	0.150	0.160	0.167
500		0.020	0.038	0.069	0.073	0.080	0.086	0.091	0.097	0.102
1000		0.015	0.030	0.054	0.057	0.061	0.065	0.068	0.073	0.076

COMPOUND (SEPARATED) POISSON PARTIAL DURATION MODEL WITH WEIBULL DISTRIBUTIONS AS MARGINALS

TABLE A.103 BIAS OF SELECTED QUANTILES: (M1 = 1.0 M2 = 1.5 CV1= 0.5 CV2= 3.0 L1 = 1.0 L2 = 0.5)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	0.879	1.599	2.103	3.598	8.282	13.366	19.907	30.973	41.300
10		-0.039	0.021	0.286	0.393	0.317	0.379	0.492	0.700	0.897
20		-0.037	-0.014	0.116	0.205	0.151	0.179	0.226	0.304	0.371
30		-0.024	-0.010	0.077	0.125	0.067	0.082	0.108	0.150	0.185
50		-0.009	-0.009	0.037	0.078	0.040	0.052	0.069	0.095	0.115
100		-0.004	-0.003	0.018	0.045	0.025	0.034	0.045	0.061	0.073
200		0.003	0.003	0.013	0.035	0.025	0.029	0.033	0.040	0.044
500		-0.001	0.000	0.003	0.008	0.005	0.007	0.008	0.011	0.013
1000		-0.001	-0.001	0.000	0.002	0.002	0.003	0.004	0.006	0.007

TABLE A.104 RMSE OF SELECTED QUANTILES: (M1 = 1.0 M2 = 1.5 CV1= 0.5 CV2= 3.0 L1 = 1.0 L2 = 0.5)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	0.879	1.599	2.103	3.598	8.282	13.366	19.907	30.973	41.300
10		0.345	0.371	1.052	1.693	2.214	2.871	3.750	5.275	6.730
20		0.250	0.167	0.529	0.974	1.183	1.375	1.597	1.943	2.252
30		0.197	0.130	0.377	0.714	0.828	0.924	1.032	1.191	1.321
50		0.144	0.094	0.236	0.519	0.610	0.676	0.748	0.851	0.933
100		0.101	0.066	0.144	0.376	0.441	0.483	0.529	0.591	0.638
200		0.069	0.045	0.086	0.265	0.299	0.319	0.341	0.369	0.390
500		0.047	0.030	0.050	0.169	0.190	0.203	0.217	0.234	0.246
1000		0.032	0.021	0.033	0.116	0.127	0.136	0.145	0.156	0.164

COMPOUND (SEPARATED) POISSON PARTIAL DURATION MODEL WITH WEIBULL DISTRIBUTIONS AS MARGINALS

TABLE A.105 BIAS OF SELECTED QUANTILES: (M1 = 1.0 M2 = 1.5 CV1= 0.5 CV2= 3.0 L1 = 1.3 L2 = 0.7)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	1.084	1.781	2.493	5.029	10.554	16.339	23.632	35.764	46.949
10		-0.028	0.063	0.333	0.231	0.226	0.287	0.377	0.530	0.668
20		-0.023	0.003	0.169	0.102	0.107	0.141	0.183	0.246	0.298
30		-0.006	-0.001	0.120	0.051	0.040	0.052	0.068	0.093	0.113
50		-0.006	0.000	0.090	0.033	0.028	0.037	0.048	0.064	0.077
100		-0.001	0.003	0.058	0.028	0.035	0.044	0.052	0.064	0.072
200		0.002	0.001	0.034	0.013	0.015	0.017	0.020	0.024	0.028
500		-0.001	0.001	0.013	0.006	0.008	0.010	0.011	0.013	0.015
1000		-0.001	-0.001	0.003	-0.004	-0.004	-0.004	-0.004	-0.003	-0.003

TABLE A.106 RMSE OF SELECTED QUANTILES: (M1 = 1.0 M2 = 1.5 CV1= 0.5 CV2= 3.0 L1 = 1.3 L2 = 0.7)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	1.084	1.781	2.493	5.029	10.554	16.339	23.632	35.764	46.949
10		0.230	0.443	1.105	1.290	1.644	2.066	2.614	3.531	4.374
20		0.154	0.190	0.623	0.785	0.937	1.080	1.249	1.512	1.744
30		0.125	0.135	0.467	0.610	0.696	0.763	0.835	0.938	1.019
50		0.097	0.099	0.350	0.479	0.540	0.589	0.644	0.722	0.785
100		0.068	0.068	0.237	0.348	0.385	0.416	0.446	0.487	0.516
200		0.049	0.049	0.163	0.254	0.276	0.293	0.311	0.334	0.351
500		0.031	0.028	0.084	0.146	0.160	0.172	0.183	0.197	0.207
1000		0.021	0.020	0.055	0.101	0.103	0.115	0.122	0.130	0.136

COMPOUND (SEPARATED) POISSON PARTIAL DURATION MODEL WITH WEIBULL DISTRIBUTIONS AS MARGINALS

TABLE A.107 BIAS OF SELECTED QUANTILES: (M1 = 1.0 M2 = 1.5 CV1= 0.5 CV2= 3.0 L1 = 2.0 L2 = 1.0)

SAMPLE SIZE	T = 2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q = 1.343	2.055	3.497	6.864	13.324	19.881	27.998	41.292	53.407
10	-0.014	0.143	0.271	0.209	0.251	0.315	0.397	0.530	0.648
20	-0.011	0.051	0.110	0.056	0.068	0.089	0.113	0.148	0.175
30	-0.009	0.018	0.061	0.018	0.025	0.036	0.048	0.065	0.078
50	-0.005	0.012	0.049	0.020	0.027	0.036	0.045	0.058	0.068
100	-0.001	0.005	0.025	0.012	0.018	0.023	0.028	0.035	0.041
200	0.001	0.005	0.014	0.007	0.009	0.011	0.013	0.016	0.018
500	0.000	0.001	0.005	0.004	0.005	0.007	0.008	0.009	0.011
1000	-0.001	-0.001	-0.003	-0.003	-0.003	-0.002	-0.001	-0.001	0.000

TABLE A.108 RMSE OF SELECTED QUANTILES: (M1 = 1.0 M2 = 1.5 CV1= 0.5 CV2= 3.0 L1 = 2.0 L2 = 1.0)

SAMPLE SIZE	T = 2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q = 1.343	2.055	3.497	6.864	13.324	19.881	27.998	41.292	53.407
10	0.150	0.576	1.062	1.211	1.553	1.950	2.502	3.537	4.605
20	0.101	0.303	0.632	0.704	0.806	0.894	0.990	1.122	1.227
30	0.082	0.178	0.457	0.521	0.585	0.635	0.687	0.756	0.808
50	0.065	0.128	0.370	0.424	0.475	0.516	0.557	0.611	0.651
100	0.045	0.080	0.273	0.306	0.335	0.358	0.381	0.410	0.431
200	0.031	0.051	0.185	0.205	0.225	0.240	0.254	0.273	0.286
500	0.020	0.029	0.112	0.122	0.134	0.144	0.153	0.164	0.172
1000	0.015	0.023	0.089	0.094	0.102	0.108	0.114	0.122	0.127

SIMPLE (MIXED) POISSON PARTIAL DURATION MODEL WITH WEIBULL DISTRIBUTION AS MARGINAL

TABLE A.109 BIAS OF SELECTED QUANTILES: (M1 = 1.0 M2 = 1.5 CV1= 0.5 CV2= 1.0 L1 = 1.0 L2 = 0.5)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	1.027	1.791	2.433	3.417	4.813	5.861	6.904	8.281	9.321
10		0.028	0.117	0.042	-0.119	-0.254	-0.316	-0.361	-0.405	-0.431
20		0.023	0.125	0.052	-0.109	-0.245	-0.309	-0.355	-0.400	-0.426
30		0.029	0.129	0.055	-0.107	-0.245	-0.308	-0.355	-0.401	-0.428
50		0.041	0.139	0.064	-0.100	-0.238	-0.303	-0.350	-0.396	-0.424
100		0.043	0.146	0.071	-0.093	-0.232	-0.297	-0.345	-0.391	-0.419
200		0.049	0.154	0.079	-0.086	-0.226	-0.291	-0.339	-0.386	-0.414
500		0.044	0.150	0.076	-0.088	-0.228	-0.293	-0.341	-0.388	-0.416
1000		0.044	0.150	0.077	-0.088	-0.228	-0.293	-0.341	-0.388	-0.416

TABLE A.110 RMSE OF SELECTED QUANTILES: (M1 = 1.0 M2 = 1.5 CV1= 0.5 CV2= 1.0 L1 = 1.0 L2 = 0.5)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	1.027	1.791	2.433	3.417	4.813	5.861	6.904	8.281	9.321
10		0.343	0.339	0.331	0.327	0.382	0.422	0.455	0.489	0.510
20		0.246	0.260	0.239	0.241	0.317	0.365	0.403	0.442	0.465
30		0.203	0.225	0.197	0.205	0.294	0.347	0.387	0.429	0.453
50		0.152	0.197	0.158	0.167	0.269	0.326	0.370	0.413	0.439
100		0.117	0.177	0.126	0.133	0.249	0.309	0.355	0.400	0.427
200		0.089	0.169	0.108	0.109	0.234	0.298	0.344	0.390	0.418
500		0.066	0.157	0.090	0.099	0.232	0.296	0.343	0.390	0.418
1000		0.056	0.154	0.083	0.093	0.230	0.295	0.342	0.389	0.416

SIMPLE (MIXED) POISSON PARTIAL DURATION MODEL WITH WEIBULL DISTRIBUTION AS MARGINAL

TABLE A.111 BIAS OF SELECTED QUANTILES: (M1 = 1.0 M2 = 1.5 CV1= 0.5 CV2= 1.0 L1 = 1.3 L2 = 0.7)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	1.238	2.029	2.857	3.920	5.318	6.365	7.409	8.785	9.826
10		0.081	0.111	-0.024	-0.166	-0.273	-0.326	-0.366	-0.405	-0.429
20		0.073	0.116	-0.017	-0.158	-0.267	-0.320	-0.360	-0.400	-0.424
30		0.090	0.122	-0.015	-0.159	-0.269	-0.323	-0.364	-0.405	-0.430
50		0.091	0.128	-0.007	-0.151	-0.262	-0.316	-0.357	-0.398	-0.422
100		0.094	0.137	0.002	-0.143	-0.254	-0.308	-0.349	-0.390	-0.415
200		0.098	0.137	0.001	-0.145	-0.256	-0.311	-0.352	-0.394	-0.418
500		0.095	0.138	0.003	-0.142	-0.254	-0.308	-0.350	-0.391	-0.416
1000		0.095	0.136	0.000	-0.145	-0.256	-0.311	-0.352	-0.393	-0.418

TABLE A.112 RMSE OF SELECTED QUANTILES: (M1 = 1.0 M2 = 1.5 CV1= 0.5 CV2= 1.0 L1 = 1.3 L2 = 0.7)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	1.238	2.029	2.857	3.920	5.318	6.365	7.409	8.785	9.826
10		0.291	0.306	0.277	0.305	0.367	0.406	0.437	0.470	0.490
20		0.202	0.228	0.191	0.238	0.315	0.360	0.395	0.431	0.453
30		0.179	0.203	0.157	0.215	0.302	0.350	0.387	0.425	0.449
50		0.152	0.182	0.126	0.191	0.284	0.334	0.372	0.411	0.435
100		0.127	0.164	0.088	0.164	0.265	0.317	0.357	0.397	0.422
200		0.117	0.153	0.066	0.157	0.262	0.316	0.356	0.397	0.422
500		0.102	0.143	0.038	0.147	0.256	0.310	0.351	0.393	0.417
1000		0.098	0.139	0.026	0.147	0.257	0.311	0.352	0.394	0.419

SIMPLE (MIXED) POISSON PARTIAL DURATION MODEL WITH WEIBULL DISTRIBUTION AS MARGINAL

TABLE A.113 BIAS OF SELECTED QUANTILES: (M1 = 1.0 M2 = 1.5 CV1= 0.5 CV2= 1.0 L1 = 2.0 L2 = 1.0)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	1.500	2.371	3.377	4.455	5.853	6.900	7.944	9.320	10.361
10		0.131	0.079	-0.090	-0.206	-0.297	-0.344	-0.380	-0.417	-0.440
20		0.122	0.073	-0.095	-0.210	-0.302	-0.348	-0.385	-0.422	-0.444
30		0.126	0.074	-0.096	-0.212	-0.304	-0.352	-0.388	-0.425	-0.448
50		0.131	0.081	-0.089	-0.205	-0.298	-0.346	-0.382	-0.420	-0.443
100		0.135	0.085	-0.085	-0.202	-0.295	-0.343	-0.380	-0.417	-0.440
200		0.137	0.088	-0.083	-0.200	-0.293	-0.341	-0.378	-0.416	-0.439
500		0.137	0.088	-0.082	-0.199	-0.292	-0.340	-0.377	-0.415	-0.438
1000		0.136	0.087	-0.083	-0.200	-0.293	-0.341	-0.378	-0.416	-0.439

TABLE A.114 RMSE OF SELECTED QUANTILES: (M1 = 1.0 M2 = 1.5 CV1= 0.5 CV2= 1.0 L1 = 2.0 L2 = 1.0)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	1.500	2.371	3.377	4.455	5.853	6.900	7.944	9.320	10.361
10		0.258	0.250	0.236	0.291	0.356	0.395	0.425	0.458	0.478
20		0.195	0.182	0.181	0.255	0.332	0.374	0.407	0.442	0.463
30		0.175	0.150	0.154	0.241	0.323	0.367	0.402	0.438	0.460
50		0.163	0.133	0.133	0.225	0.311	0.356	0.392	0.428	0.451
100		0.152	0.116	0.111	0.213	0.302	0.349	0.385	0.422	0.445
200		0.145	0.102	0.096	0.205	0.296	0.344	0.380	0.418	0.441
500		0.140	0.094	0.087	0.201	0.294	0.341	0.378	0.416	0.439
1000		0.137	0.090	0.086	0.201	0.294	0.342	0.379	0.416	0.440

SIMPLE (MIXED) POISSON PARTIAL DURATION MODEL WITH WEIBULL DISTRIBUTION AS MARGINAL

TABLE A.115 BIAS OF SELECTED QUANTILES: (M1 = 1.0 M2 = 1.5 CV1= 0.5 CV2= 1.5 L1 = 1.0 L2 = 0.5)

SAMPLE SIZE	T = 2. Q = 0.966	T = 5. Q = 1.722	T = 10. Q = 2.384	T = 20. Q = 3.857	T = 50. Q = 6.365	T = 100. Q = 8.486	T = 200. Q = 10.780	T = 500. Q = 14.058	T = 1000. Q = 16.711
10	-0.007	0.233	0.204	-0.070	-0.290	-0.385	-0.450	-0.512	-0.546
20	-0.026	0.238	0.213	-0.064	-0.288	-0.385	-0.452	-0.516	-0.552
30	-0.020	0.240	0.211	-0.068	-0.293	-0.391	-0.459	-0.524	-0.560
50	-0.011	0.253	0.224	-0.059	-0.287	-0.386	-0.455	-0.521	-0.558
100	-0.009	0.262	0.233	-0.052	-0.282	-0.382	-0.452	-0.518	-0.556
200	-0.004	0.272	0.244	-0.043	-0.275	-0.376	-0.447	-0.513	-0.551
500	-0.009	0.268	0.239	-0.047	-0.278	-0.379	-0.449	-0.516	-0.554
1000	-0.010	0.268	0.240	-0.046	-0.278	-0.379	-0.449	-0.516	-0.554

TABLE A.116 RMSE OF SELECTED QUANTILES: (M1 = 1.0 M2 = 1.5 CV1= 0.5 CV2= 1.5 L1 = 1.0 L2 = 0.5)

SAMPLE SIZE	T = 2. Q = 0.966	T = 5. Q = 1.722	T = 10. Q = 2.384	T = 20. Q = 3.857	T = 50. Q = 6.365	T = 100. Q = 8.486	T = 200. Q = 10.780	T = 500. Q = 14.058	T = 1000. Q = 16.711
10	0.403	0.515	0.554	0.454	0.486	0.531	0.571	0.613	0.639
20	0.292	0.402	0.416	0.313	0.388	0.454	0.507	0.559	0.590
30	0.242	0.356	0.358	0.256	0.360	0.436	0.494	0.551	0.584
50	0.178	0.323	0.315	0.199	0.329	0.414	0.476	0.537	0.572
100	0.133	0.299	0.282	0.145	0.304	0.396	0.463	0.526	0.563
200	0.091	0.291	0.269	0.105	0.287	0.383	0.452	0.517	0.555
500	0.060	0.276	0.251	0.079	0.283	0.382	0.452	0.518	0.556
1000	0.043	0.272	0.245	0.063	0.280	0.380	0.450	0.517	0.555

SIMPLE (MIXED) POISSON PARTIAL DURATION MODEL WITH WEIBULL DISTRIBUTION AS MARGINAL

TABLE A.117 BIAS OF SELECTED QUANTILES: (M1 = 1.0 M2 = 1.5 CV1= 0.5 CV2= 1.5 L1 = 1.3 L2 = 0.7)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	1.175	1.951	2.961	4.717	7.363	9.574	11.950	15.327	18.049
10		0.083	0.262	0.094	-0.155	-0.327	-0.406	-0.463	-0.518	-0.550
20		0.066	0.262	0.096	-0.154	-0.328	-0.409	-0.467	-0.524	-0.556
30		0.085	0.268	0.095	-0.159	-0.335	-0.417	-0.476	-0.533	-0.566
50		0.085	0.277	0.106	-0.149	-0.327	-0.409	-0.469	-0.526	-0.560
100		0.088	0.289	0.118	-0.139	-0.319	-0.402	-0.462	-0.521	-0.555
200		0.093	0.289	0.115	-0.142	-0.322	-0.405	-0.466	-0.524	-0.559
500		0.089	0.290	0.117	-0.140	-0.320	-0.403	-0.464	-0.523	-0.557
1000		0.088	0.287	0.114	-0.143	-0.323	-0.406	-0.466	-0.525	-0.559

TABLE A.118 RMSE OF SELECTED QUANTILES: (M1 = 1.0 M2 = 1.5 CV1= 0.5 CV2= 1.5 L1 = 1.3 L2 = 0.7)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	1.175	1.951	2.961	4.717	7.363	9.574	11.950	15.327	18.049
10		0.364	0.500	0.428	0.391	0.458	0.508	0.549	0.591	0.617
20		0.246	0.391	0.300	0.286	0.391	0.455	0.504	0.554	0.583
30		0.215	0.361	0.253	0.254	0.377	0.447	0.500	0.552	0.583
50		0.177	0.337	0.215	0.218	0.355	0.429	0.484	0.539	0.571
100		0.141	0.320	0.177	0.179	0.333	0.412	0.470	0.527	0.560
200		0.123	0.306	0.152	0.165	0.330	0.411	0.470	0.528	0.562
500		0.102	0.296	0.131	0.149	0.323	0.405	0.465	0.524	0.558
1000		0.094	0.290	0.121	0.147	0.324	0.407	0.467	0.526	0.560

SIMPLE (MIXED) POISSON PARTIAL DURATION MODEL WITH WEIBULL DISTRIBUTION AS MARGINAL

TABLE A.119 BIAS OF SELECTED QUANTILES: (M1 = 1.0 M2 = 1.5 CV1= 0.5 CV2= 1.5 L1 = 2.0 L2 = 1.0)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	1.438	2.312	3.792	5.686	8.470	10.771	13.231	16.709	19.501
10		0.192	0.248	-0.038	-0.231	-0.374	-0.443	-0.494	-0.545	-0.575
20		0.176	0.235	-0.049	-0.242	-0.384	-0.453	-0.504	-0.555	-0.585
30		0.182	0.234	-0.052	-0.245	-0.388	-0.458	-0.510	-0.561	-0.591
50		0.188	0.243	-0.045	-0.239	-0.383	-0.453	-0.505	-0.557	-0.587
100		0.193	0.251	-0.038	-0.234	-0.379	-0.450	-0.502	-0.554	-0.585
200		0.196	0.253	-0.037	-0.233	-0.378	-0.449	-0.502	-0.554	-0.585
500		0.195	0.254	-0.036	-0.232	-0.377	-0.448	-0.501	-0.553	-0.584
1000		0.193	0.253	-0.037	-0.233	-0.379	-0.449	-0.502	-0.554	-0.585

TABLE A.120 RMSE OF SELECTED QUANTILES: (M1 = 1.0 M2 = 1.5 CV1= 0.5 CV2= 1.5 L1 = 2.0 L2 = 1.0)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	1.438	2.312	3.792	5.686	8.470	10.771	13.231	16.709	19.501
10		0.359	0.444	0.318	0.360	0.448	0.500	0.542	0.585	0.610
20		0.270	0.346	0.224	0.308	0.420	0.480	0.527	0.573	0.601
30		0.245	0.308	0.179	0.287	0.410	0.474	0.523	0.572	0.601
50		0.229	0.293	0.148	0.269	0.398	0.464	0.514	0.564	0.594
100		0.216	0.278	0.110	0.251	0.387	0.456	0.507	0.558	0.589
200		0.206	0.266	0.079	0.241	0.382	0.452	0.504	0.556	0.587
500		0.199	0.259	0.056	0.235	0.379	0.449	0.502	0.554	0.585
1000		0.196	0.255	0.050	0.235	0.379	0.450	0.503	0.555	0.586

SIMPLE (MIXED) POISSON PARTIAL DURATION MODEL WITH WEIBULL DISTRIBUTION AS MARGINAL

TABLE A.121 BIAS OF SELECTED QUANTILES: (M1 = 1.0 M2 = 1.5 CV1= 0.5 CV2= 3.0 L1 = 1.0 L2 = 0.5)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	0.879	1.599	2.103	3.598	8.282	13.366	19.907	30.973	41.300
10		-0.160	0.431	0.727	0.435	-0.093	-0.286	-0.406	-0.509	-0.562
20		-0.216	0.425	0.719	0.415	-0.124	-0.322	-0.446	-0.554	-0.610
30		-0.213	0.419	0.695	0.381	-0.155	-0.352	-0.476	-0.582	-0.638
50		-0.214	0.433	0.709	0.387	-0.156	-0.356	-0.482	-0.589	-0.645
100		-0.215	0.442	0.717	0.390	-0.158	-0.360	-0.486	-0.594	-0.651
200		-0.214	0.456	0.734	0.403	-0.151	-0.355	-0.483	-0.592	-0.650
500		-0.222	0.446	0.721	0.391	-0.160	-0.362	-0.489	-0.598	-0.655
1000		-0.223	0.447	0.722	0.392	-0.159	-0.362	-0.489	-0.598	-0.655

TABLE A.122 RMSE OF SELECTED QUANTILES: (M1 = 1.0 M2 = 1.5 CV1= 0.5 CV2= 3.0 L1 = 1.0 L2 = 0.5)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	0.879	1.599	2.103	3.598	8.282	13.366	19.907	30.973	41.300
10		0.491	0.857	1.307	1.163	0.833	0.791	0.795	0.819	0.839
20		0.396	0.655	1.008	0.790	0.504	0.527	0.580	0.644	0.682
30		0.350	0.578	0.888	0.645	0.403	0.471	0.549	0.629	0.675
50		0.301	0.528	0.825	0.554	0.320	0.425	0.522	0.614	0.665
100		0.265	0.494	0.779	0.481	0.252	0.394	0.506	0.606	0.660
200		0.240	0.481	0.765	0.449	0.205	0.372	0.492	0.598	0.654
500		0.232	0.458	0.735	0.413	0.185	0.370	0.493	0.601	0.657
1000		0.229	0.452	0.729	0.402	0.171	0.365	0.491	0.599	0.656

SIMPLE (MIXED) POISSON PARTIAL DURATION MODEL WITH WEIBULL DISTRIBUTION AS MARGINAL

TABLE A.123 BIAS OF SELECTED QUANTILES: (M1 = 1.0 M2 = 1.5 CV1= 0.5 CV2= 3.0 L1 = 1.3 L2 = 0.7)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	1.084	1.781	2.493	5.029	10.554	16.339	23.632	35.764	46.949
10		-0.016	0.589	0.734	0.193	-0.193	-0.348	-0.449	-0.539	-0.587
20		-0.057	0.571	0.707	0.161	-0.229	-0.385	-0.489	-0.582	-0.631
30		-0.038	0.571	0.688	0.137	-0.253	-0.410	-0.513	-0.605	-0.654
50		-0.042	0.583	0.704	0.148	-0.246	-0.405	-0.510	-0.603	-0.653
100		-0.041	0.599	0.722	0.159	-0.241	-0.402	-0.508	-0.602	-0.653
200		-0.037	0.595	0.712	0.150	-0.250	-0.410	-0.516	-0.610	-0.660
500		-0.043	0.594	0.712	0.150	-0.250	-0.410	-0.516	-0.610	-0.661
1000		-0.045	0.589	0.705	0.144	-0.254	-0.414	-0.519	-0.613	-0.664

TABLE A.124 RMSE OF SELECTED QUANTILES: (M1 = 1.0 M2 = 1.5 CV1= 0.5 CV2= 3.0 L1 = 1.3 L2 = 0.7)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	1.084	1.781	2.493	5.029	10.554	16.339	23.632	35.764	46.949
10		0.445	0.942	1.209	0.801	0.660	0.670	0.699	0.737	0.763
20		0.299	0.746	0.933	0.500	0.428	0.497	0.564	0.633	0.673
30		0.250	0.699	0.853	0.412	0.387	0.481	0.559	0.635	0.678
50		0.200	0.665	0.810	0.341	0.338	0.451	0.539	0.622	0.668
100		0.145	0.642	0.776	0.269	0.290	0.424	0.522	0.611	0.660
200		0.109	0.619	0.743	0.218	0.276	0.422	0.523	0.614	0.664
500		0.075	0.603	0.723	0.178	0.260	0.415	0.519	0.612	0.662
1000		0.062	0.593	0.710	0.158	0.259	0.416	0.521	0.614	0.664

SIMPLE (MIXED) POISSON PARTIAL DURATION MODEL WITH WEIBULL DISTRIBUTION AS MARGINAL

TABLE A.125 BIAS OF SELECTED QUANTILES: (M1 = 1.0 M2 = 1.5 CV1= 0.5 CV2= 3.0 L1 = 2.0 L2 = 1.0)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	1.343	2.055	3.497	6.864	13.324	19.881	27.998	41.292	53.407
10		0.222	0.738	0.460	-0.016	-0.315	-0.442	-0.529	-0.609	-0.652
20		0.191	0.698	0.418	-0.051	-0.346	-0.473	-0.559	-0.637	-0.680
30		0.198	0.690	0.402	-0.068	-0.363	-0.488	-0.574	-0.652	-0.695
50		0.203	0.700	0.410	-0.063	-0.360	-0.486	-0.573	-0.651	-0.694
100		0.210	0.714	0.420	-0.058	-0.357	-0.485	-0.573	-0.652	-0.695
200		0.210	0.712	0.416	-0.061	-0.361	-0.489	-0.576	-0.655	-0.698
500		0.209	0.714	0.418	-0.060	-0.360	-0.488	-0.575	-0.655	-0.698
1000		0.207	0.711	0.416	-0.062	-0.361	-0.489	-0.576	-0.656	-0.699

TABLE A.126 RMSE OF SELECTED QUANTILES: (M1 = 1.0 M2 = 1.5 CV1= 0.5 CV2= 3.0 L1 = 2.0 L2 = 1.0)

SAMPLE SIZE	T =	2.	5.	10.	20.	50.	100.	200.	500.	1000.
	Q =	1.343	2.055	3.497	6.864	13.324	19.881	27.998	41.292	53.407
10		0.478	1.027	0.832	0.528	0.524	0.577	0.627	0.681	0.713
20		0.335	0.844	0.623	0.352	0.441	0.529	0.598	0.664	0.702
30		0.297	0.783	0.536	0.273	0.416	0.519	0.595	0.666	0.706
50		0.270	0.765	0.506	0.231	0.399	0.509	0.587	0.661	0.702
100		0.247	0.748	0.470	0.167	0.377	0.496	0.580	0.657	0.699
200		0.228	0.728	0.441	0.125	0.370	0.494	0.579	0.657	0.700
500		0.216	0.720	0.428	0.090	0.363	0.490	0.576	0.655	0.699
1000		0.211	0.714	0.421	0.080	0.363	0.490	0.577	0.656	0.699

Appendix B

Analysis of Observed Data

Table B.1 Pertinent Data for Observed Partial Duration Series in Louisiana

Station Code	USGS Station No	Drainage $A(km^2)$	Years of Obs.	Period of Obs.	USGS Thr. $Q(m^3/s)$
1	02491500	2534	66	1922-87	198
2	07376000	632	47	1941-87	57
3	07376500	205	44	1944-87	48
4	07375000	103	44	1944-87	37
5	07377000	264	39	1949-87	226
6	02492000	3105	50	1938-87	226
7	07378500	3277	49	1939-87	57
8	07375500	1654	49	1939-87	226
9	07378000	727	44	1944-87	226
10	07377500	371	45	1943-87	85
11	08013500	1928	49	1939-87	226
12	08014500	1306	48	1940-87	85
13	08014200	241	37	1950-86	14
14	08013000	1277	44	1944-87	113
15	08016400	379	39	1946-84	45
16	08016600	210	38	1946-83	57
17	08031000	212	34	1953-86	25
18	08030000	177	32	1952-83	28
19	07351500	169	49	1939-87	85
20	08029500	328	36	1952-87	31
21	07352000	394	47	1941-87	28
22	07373000	131	46	1942-87	27
23	07371500	909	49	1939-87	85
24	07366000	1183	43	1941-83	113
25	07352500	1083	43	1941-83	85
26	07349500	1398	49	1939-87	85
27	07362100	986	49	1939-87	68

Table B.2 Poisson Test on Annually Homogeneous Observed PDS ($\alpha = 10\%$)

Station Code	Threshold Q USGS	Threshold Q Poisson	Poisson Mean-λ	Poisson Ratio-R	Critical Ratio-R_c	Poissonian Process
1	198	228	1.41	1.20	1.23	Accepted
2	57	65	2.26	1.27	1.27	Accepted
3	48	71	1.27	1.08	1.28	Accepted
4	37	57	1.64	1.14	1.28	Accepted
5	226	255	1.92	1.26	1.30	Accepted
6	226	498	1.00	1.48	1.26	Rejected
7	57	665	1.00	1.55	1.27	Rejected
8	226	380	1.00	1.39	1.27	Rejected
9	226	283	0.99	1.46	1.28	Rejected
10	85	157	1.00	1.51	1.28	Rejected
11	226	226	1.73	1.24	1.26	Accepted
12	85	127	2.25	1.06	1.27	Accepted
13	14	22	1.78	1.25	1.31	Accepted
14	113	141	2.34	1.18	1.28	Accepted
15	45	45	3.00	1.21	1.30	Accepted
16	57	75	1.13	1.17	1.31	Accepted
17	25	29	1.38	1.32	1.33	Accepted
18	28	28	2.13	1.11	1.34	Accepted
19	85	85	1.42	1.07	1.27	Accepted
20	31	54	1.00	2.22	1.32	Rejected
21	28	57	1.06	1.28	1.27	Accepted
22	27	57	1.04	1.12	1.27	Accepted
23	85	145	1.12	1.26	1.27	Accepted
24	113	129	1.05	1.24	1.29	Accepted
25	85	100	1.00	1.81	1.28	Rejected
26	85	113	1.00	1.38	1.27	Rejected
27	68	120	1.00	1.42	1.27	Rejected

Table B.3 Mod. Mann-Whitney U Test on Mixed Populations ($\alpha = 10\%$)

Station Code	Threshold Q USGS	Threshold Q Poisson	Std Mod U Z_{U_c}	Mixed Populations	Hydrologi c Region	CV at Poi. Q
1	198	228	0.784	None	SE	1.13
2	57	65	1.547	None	SE	1.20
3	48	71	0.875	None	SE	1.12
4	37	57	0.916	None	SE	1.25
5	226	255	0.363	None	SE	1.09
11	226	226	0.303	None	SW	1.21
12	85	127	2.138	Exist	SW	1.91
13	14	22	0.815	None	SW	1.47
14	113	141	1.789	Exist	SW	1.28
15	45	45	2.500	Exist	SW	1.41
16	57	75	2.292	Exist	SW	1.07
17	25	29	1.137	None	SW	1.32
18	28	28	1.299	None	SW	1.09
19	85	85	0.488	None	SW	1.22
21	28	57	0.692	None	NW	1.27
22	27	57	0.429	None	NW	1.36
23	85	145	0.954	None	NW	1.32
24	113	129	0.082	None	NW	1.69

Critical Values of Z_{U_c} for Different α's:

α	$Z_{1-\alpha/2}$	α	$Z_{1-\alpha/2}$	α	$Z_{1-\alpha/2}$
5%	1.960	10%	1.645	15%	1.440

Hydrologic Regions: Ref. Naghavi, et. al., 1989.

Table B.4 Exponential Test on Annually Homogeneous Observed PDS ($\alpha = 5\%$)

Station	Threshold Q			Kolgomorov Test		Cramer-Von Test	
Code	USGS	Poisson	Expon	D	Remark	w^2	Remark
1	198	228	228	1.026	Pass	0.188	Pass
2	57	65	67	0.877	Pass	0.211	Pass
3	48	71	71	0.765	Pass	0.084	Pass
4	37	57	57	0.876	Pass	0.205	Pass
5	226	255	255	0.959	Pass	0.187	Pass
11	226	226	226	0.963	Pass	0.096	Pass
12	85	127	259*	1.418	Fail	0.546	Fail
13	14	22	51	1.009	Pass	0.209	Pass
14	113	141	146	0.983	Pass	0.208	Pass
15	45	45	57	1.079	Pass	0.184	Pass
16	57	75	78*	1.386	Fail	0.329	Fail
17	25	29	32*	1.119	Fail	0.233	Fail
18	28	28	28	0.722	Pass	0.049	Pass
19	85	85	88	0.946	Pass	0.169	Pass
21	28	57	62*	1.504	Fail	0.459	Fail
22	27	57	59*	1.169	Fail	0.595	Fail
23	85	145	151	1.089	Pass	0.194	Pass
24	113	129	140*	1.351	Fail	0.496	Fail

Critical Values of EDF Test Statistics (Stephens, 1974):

At $\alpha = 5\%$		At $\alpha = 10\%$		At $\alpha = 15\%$	
D	W^2	D	W^2	D	W^2
1.094	0.224	0.990	0.177	0.926	0.149

'*' At this Q both EDF tests failed as $\lambda < 1$ flood event/yr.

Table B.5 Fitting Simple Models To Annually Homogeneous Observed PDS

Station Code	Threshold Q			SRMSE		Hydrologic Region
	USGS	Poisson	Expon	Exponential	Weibull	
1	198	228	228	0.259	0.246	SE
2	57	65	67	0.197	0.126	SE
3	48	71	71	0.127	0.095	SE
4	37	57	57	0.329	0.262	SE
5	226	255	255	0.164	0.154	SE
11	226	226	226	0.234	0.168	SW
12	85	127	-	-	0.797	SW
13	14	22	51	0.590	0.501	SW
14	113	141	146	0.194	0.116	SW
15	45	45	57	0.261	0.193	SW
16	57	75	-	-	0.193	SW
17	25	29	-	-	0.191	SW
18	28	28	28	0.165	0.160	SW
19	85	85	88	0.173	0.138	SW
21	28	57	-	-	0.107	NW
22	27	57	-	-	0.152	NW
23	85	145	151	0.314	0.188	NW
24	113	129	-	-	0.683	NW

Exponential admissible threshold Q was used to fit the exponential model.
Poisson admissible threshold Q was used to fit the Weibull model.
'-' Both EDF tests on exponential hypothesis failed.

Table B.6 Fitting Compound Models To Annually Nonhomogeneous Obs. PDS

Station	Threshold Q			SRMSE		Hydrologic
Code	USGS	Poisson	Expon	Exponential	Weibull	Region
1	198	228	228	0.253	0.242	SE
2	57	65	67	0.194	0.127	SE
3	48	71	71	0.114	0.094	SE
4	37	57	57	0.327	0.260	SE
5	226	255	255	0.160	0.178	SE
11	226	226	226	0.215	0.127	SW
12	85	127	-	-	0.678	SW
13	14	22	51	0.549	0.423	SW
14	113	141	146	0.163	0.122	SW
15	45	45	57	0.195	0.172	SW
16	57	75	-	-	0.210	SW
17	25	29	-	-	0.205	SW
18	28	28	28	0.146	0.163	SW
19	85	85	88	0.147	0.162	SW
21	28	57	-	-	0.106	NW
22	27	57	-	-	0.151	NW
23	85	145	151	0.314	0.188	NW
24	113	129	-	-	0.679	NW

Exponential admissible threshold Q was used to fit the exponential model.
Poisson admissible threshold Q was used to fit the Weibull model.
'-' Both EDF tests on exponential hypothesis failed.

Vita

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Title of Dissertation: Comparative Evaluation of Exact Extreme Value Stochastic Flood Model for Mixed Populations

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